Integrating dynamic software into geometry courses at middle school, high school, and college levels: Ten lesson plan and instruction materials units incorporating Geometer’s Sketchpad Version 4.07

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Introduction

The NCTM Principles and Standards (2000) recommends extensive use of available technologies in the teaching and learning of mathematics at all levels, from elementary school to college. In the content areas of geometry and measurement, software tools such as Geometer’s Sketchpad are becoming more widely available in classrooms, especially at the middle school, high school, and college levels. However, challenges to use of these technological tools still exist, and it is critical to integrate research-based best practices for classroom pedagogy, thorough understanding of the geometric concepts, and the ability to use these tools to facilitate active and engaged learning by students.

This paper provides an overview of a senior-level honors project completed by two mathematics education majors. Working under the direction of their Math 321 (Geometry) professor, these two students chose a project that connected that course with a concurrent course, Math 380 (Methods of Teaching Mathematics in Grades 6-12). The final product consists of ten lesson units that address a wide variety of topics. Some of the units would be most appropriate for a middle school or high school geometry course. Other units address topics more appropriate for an honors level high school geometry course or an advanced college level geometry course. Each unit includes a lesson plan, Sketchpad-based worksheet and solution key, and associated Sketchpad documents and scripts compatible with version 4.07. The website for all of these materials is provided in the reference section of this paper.

Insights from research: Best practices in teaching and learning mathematics

Teaching mathematics is always a challenge. Many students seem to have an aversion to mathematics, at least more so than most other subjects. Because of this, math teachers have the difficult task of teaching students who have very little desire to be in their classroom and very little motivation to learn. In part, this lack of motivation and desire relates to the different ways in which students learn. Howard Gardner (1993) developed a theory of multiple intelligences. Although some people, including most math teachers, work best in logical and spatial modes, other people prefer learning modes that emphasize discourse among peers, the ability to move their own bodies and direct the movement of objects such as polygons and circles in a dynamic fashion, and the opportunity to discuss their conjectures in writing. Experiential teaching and learning techniques build on these learning styles while also addressing logical and spatial learning modes.
One way that this experiential learning can take place is through technology. *Geometer’s Sketchpad* is one of many excellent math programs that can easily be implemented into a mathematics classroom. Through this software program, students can solve problems in a fun, interactive, and dynamic environment. Students can also take what they have learned and expand on those ideas. Although *Sketchpad* cannot easily create proofs, it can provide the grounds and basis for developing proofs and can be the visual aid needed to help some students understand concepts. These features of *Sketchpad* are consistent with the Technology Principle and the Reasoning and Proof Process Standard that are advocated by NCTM (2000).

Math teachers need to begin taking advantage of the knowledge and problem solving skills that their students already have. Instead of giving a formula to a student, give the students the tools necessary to develop the formula on their own. By allowing students to explore, with limited guidance where appropriate, students will see why mathematics is so important and why certain properties and theorems work. Students will also become more excited about class because they are given the opportunity to do more than sit and listen to a lecture. Guided explorations, especially in the context of dynamic geometry software, are consistent with best teaching practices cited in standard texts for the methods of teaching mathematics at the middle school and high school level (e.g. Sheffield and Cruikshank, 2004; Posamentier, Smith, and Stepelman, 2006). It is reasonable to extend such recommendations to the college level as well.

Although several published resources exist, including college level geometry books that teach geometry in the context of *Geometer’s Sketchpad* (e.g. Reynolds and Fenton, 2006), the two mathematics education majors were encouraged to developed some ideas for lesson units based upon ideas gleaned from two concurrent courses, an advanced course in modern geometry and a methods course for teaching mathematics in grades 6-12. These two students worked collaboratively to develop a total of ten lesson units, and these units satisfied an honors project requirement for the college.

**Lesson Unit Overviews**

The following section includes short overviews for each of the ten lesson units. Note that there are actually 11 sections because the “9 point circle” and the “Euler line” lessons are part of the same unit and both follow naturally from the lesson unit that encourages students to explore special points associated with triangles. While almost any of these lesson units could be adapted for use at the middle school, high school, or college levels, a code has been included for those levels that would be most appropriate (MS – middle school; HS – high school; C – college).

**Parallel and Perpendicular Lines [MS, HS]**

The lesson on parallel and perpendicular lines is an introduction to *Geometer’s Sketchpad*. It shows students how to navigate through the program in a structured way while also reviewing and learning properties of parallel and perpendicular lines. This lesson explores ideas of transversals, angles created by transversals and their measurement. It also teaches students to create a square on *Sketchpad* using parallel and perpendicular lines and a compass. It ends by teaching students to create scripts so they don’t have to repeat all the steps to make a simple shape like a square each time.
Polygons (Interior Angle Theorem) [MS, HS]

The lesson on polygons has students exploring the sum of the measures of the interior of polygons. From the lab, students will be able to find a pattern and create a formula for finding the sum of any n-sided polygon.

Pythagorean Theorem [MS, HS, C]

The Pythagorean Theorem is one theorem that most students can recite in a heartbeat. They all know that $a^2 + b^2 = c^2$. However, many of them have no idea why this theorem actually works. This series of three lesson plans explores the topic of the Pythagorean Theorem, attempting to show through visuals and demonstrate through a simple proof that the theorem works. The first lesson two lesson plans have the students creating squares off of the sides of right triangles. In one, the students highlight the interior of the squares and use the equation feature on Sketchpad to add the areas of the two smaller squares, showing that they always equal the larger square. The third lesson plan has students creating a large square with sides $a + b$. Students then prove the Pythagorean Theorem by creating two different equations for the area of the square, noting that they simplify to $a^2 + b^2 = c^2$.

Exploring triangles and special points [MS, HS, C]

In this lesson the student will explore some properties of triangles. Specifically, they will explore the medians, altitudes, perpendicular bisectors, and angle bisectors of triangles. Using Geometer’s Sketchpad the student will create these lines within a triangle they have also created and explore what happens to their concurrent points when the type of triangle is changed. Specifically, they will look at these concurrent points with isosceles, equilateral, acute, obtuse and right triangles. In the end, the students should save their lab for use with the Euler Line lab.

The Golden Ratio [HS, C]

In this lesson the students first discover what the golden ratio is using the Fibonacci sequence and body measurements. Then the students move on to creating the golden rectangle using Geometer’s Sketchpad. They learn how to use the custom script program in Sketchpad to help them create the golden rectangle. After the rectangle is created, the students explore why it is called the golden rectangle by exploring the many different ratios within it. Lastly, the students create the golden spiral using the golden rectangle and the custom tool program. This lesson unit is similar to the technology-linked portions of “Fibonacci Numbers and the Golden Ratio” published in a collection Mathematics: A Christian Perspective (Bradley et al., 2007).

Tessellations [HS, C]

The students learn and explore about regular and semi-regular tessellations. Using the different transformation tools (rotate, dilate, reflect) in Sketchpad, the students explore what angles create rotation tessellations and which do not. Then, they use the different shapes they have created to see which one of these tile a plane and which will not. Through this they learn which shapes will create regular tessellations and which will create semi-regular tessellations.

Projective Geometry/Perspective Drawing [HS, C]

The lesson on perspective drawing is aimed at connecting mathematics and art through the use of technology. In this lesson, students will create a drawing in one-point perspective while still needing to complete some mathematics. Students will need to use what they know
about perspective and how objects appear smaller as they move farther away in order to create this drawing. This lesson plan could easily be done without the use of Sketchpad, but the technology enhances the lesson. Not only do most students enjoy using technology, but students will also be able to explore easily how objects appear changes when a vanishing point moves. All students need to do is drag their vanishing point and all path objects will move with it.

**Euler Line [C]**

The students use their work from the exploring triangles lab in order to create the Euler line. Since the Euler Line is created from a few of the concurrent points explored earlier, the students will explore what happens to these points in a triangle. Then, they will be told what points create the Euler line. Once they actually create the Euler line, the students then explore the line within different types of triangles. The students also begin constructing the nine-point circle from the Euler line.

**Nine-point circle [C]**

The students use their knowledge of the special concurrent points in the exploring triangles lab in order to create the nine-point circle for a triangle of their choice. Once they have created the nine-point circle, the students will explore as they have done previously; they change the type of triangle they have created into an isosceles, equilateral, acute, obtuse, and right triangle in order to see what happens to the circle in these types of triangles. The students also explore how the Euler line fits into the nine-point circle.

**Fractals [C]**

The students learn about fractals by seeing a few pictures and then by building one. The students use the transformation and custom script tools in order to create a portion of the von Koch curve. Using this, they discover a few properties of fractals such as their self-similarity. The students also learn about the dimensions of fractals and learn how to calculate the dimensions of some very simple fractals. They also create the Sierpinski Triangle using Sketchpad.

**Transformation (7 cases to prove closure property for Euclidean motions) [C]**

The lesson on transformations is focused around the theorem that states, “Every plane motion can be written as the product of three or fewer reflections and conversely, three or fewer reflections can be written as one plane motion.” In the lesson, students will be focusing on the converse of the theorem. Their task will be to reflect a triangle over three lines and discover how that series of transformations can be written as one motion. There are seven different ways three lines can be drawn but each student would be responsible for looking at only one or two of those ways. The students will be required to explain how they came to their conclusion and share their results with the class. To close the lesson, students discuss the idea of orientation and why none of their transformations ended in a rotation or translation.

**Recent and future implementation of the lesson units**

Several of these lesson units are similar to ones that have previously been used successfully at the high school and college levels. In particular, students in Math 321 in Fall 2008 completed Sketchpad labs that focused on special points of triangles, the 9-point circle, and
the Euler line. After the students developed the lesson unit on transformation, the professor used excerpts from this unit to illustrate various cases that were necessary to establish the closure property for the Euclidean motions of the plane. During the 2009-2010 academic year, the students anticipate having opportunities to use some of these lesson units in their novice teaching and student teaching internships, and both students hope to make more extensive use of these materials in their own classrooms in future years.

Summary

Overall, Sketchpad is a great way to allow students to explore concepts on their own, to develop their own ideas and conjectures without being told. Lab documents provide the guidance necessary to ensure that students do reach some conclusions, but students are given the opportunity to take ownership for their work. This will not only create excitement and help motivate students but it will also develop their higher level thinking and problem solving skills.

Bibliography


Website for Sketchpad Lesson Units: http://www.trnty.edu/mathtriathlon/sketchpadindex.htm