A Tale of Two Transitions

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Many students experience difficulty making the transition from Calculus to advanced courses with an increased emphasis on the writing of proofs. This paper describes two approaches which address this problem. One of these, pursued by Northern Illinois University, involves the creation of a new course in mathematical reasoning. The second approach, adopted by Trinity Christian College, involves the redesign of a course in discrete structures. Both courses, targeted to sophomore mathematics majors, attempt to introduce sophisticated mathematical reasoning in topics which are accessible to students. An analysis of the effectiveness of each of these two approaches will be provided.

Introduction

It is not uncommon for undergraduate students to experience success in Calculus courses but then experience great difficulties in advanced courses which require the student to write and understand proofs. Such students, while often well-trained in applying mathematical concepts and solving related problems, typically have had little experience with proofs, other than those which have been presented in finished form by the instructor. What appears to be necessary then is for these students to have ample opportunity and "soaking time" to learn necessary components of mathematical proofs such as logic and set theory and to implement these components by constructing proofs of conjectures in applications which are easily accessible to them.

A course which meets these needs would fulfill the definition of a bridge or transition course between Calculus and advanced courses. Many colleges and universities do not offer such a course, choosing instead to use an elementary course in linear algebra, differential equations, or some other area of mathematics as a first course to focus on the writing and understanding of mathematical proofs. This approach usually requires limiting the time spent on the components of proof in order to cover the necessary topics in the syllabus of the underlying content course. Until recently, both Northern Illinois University and Trinity Christian College followed this approach.

During the past two years, however, both institutions decided to develop a transition course which addresses this need for students to have opportunities to focus on the components of proofs. Despite this common goal, the underlying factors related to the development of each course, the structure of each course, and the observed results of each course were unique to each institution. What follows is a narrative or tale about each of these two transition courses. Each tale addresses the nature of the institution and factors related to the development of the transition course, a description of the course, and a discussion of the observed effectiveness of the course.

The First Tale: Northern Illinois University

Northern Illinois University is a state-related institution with an enrollment of more than 20,000 students. Most undergraduate students come from the area in Illinois north of Interstate 80; many of these commute to classes from outside DeKalb County. More than half the graduates of the university are transfer students from other institutions (primarily community colleges). There are approximately 150 undergraduate majors in mathematics, primarily in the mathematics education emphasis. In a typical academic year, between 30 and 40 students, at both the graduate and undergraduate levels, will complete the certification program to teach high school or middle school mathematics.

As in many other institutions of all sizes, students at NIU frequently encounter difficulty in moving from the calculus sequence, with its typical focus on algorithmic calculations and procedures, to upper division courses (such as advanced calculus and abstract algebra) in which a deep theoretical understanding of the material is stressed. The Department of Mathematical Sciences at NIU introduced a special three credit course, MATH 280 (Introduction to Mathematical Reasoning), in 1996, in an attempt to assist students in making this challenging transition successfully. The course was intended to provide students with a detailed introduction to the special language of mathematical proof. It was envisioned that students would be given frequent opportunities to read, listen to, and evaluate mathematical arguments given by others, as well as to develop their own ability to reason in a rigorous way in mathematical settings. The formal (catalog) description of the course is below.

MATH 280. INTRODUCTION TO MATHEMATICAL REASONING (3)

> *An introduction to the idea of mathematical proof. Special emphasis will be placed on improving students' ability to construct, explain, and justify mathematical arguments. The course is intended to assist with the transition from the calculus sequence to more abstract and theoretical courses. Prerequisite: Calculus II*

This course has been presented twice (Spring and Fall 1996) by the third author to a total of 18 students. This portion of the paper constitutes a preliminary report on this experiment in learning and teaching, and its perceived outcomes. The analysis of data is longitudinal, in that the performance of those completing the course in subsequent theoretical courses was tracked, and most of the students were interviewed after completing the course to obtain their reactions to it.

During the 1993-94 academic year, the third author was on sabbatical leave at the University of California, Berkeley, where he audited a transition course similar in spirit to MATH 280. This course had been developed by the graduate students at Berkeley, and was presented in a variety of formats: lectures, small group work in class, board presentations by students, and oral examinations, among others. It was felt that such an experience might also be beneficial for students at NIU.

Historically, mathematics majors at NIU have had a low success rate in the first course in abstract algebra (mostly finite groups), and the first course in advanced calculus (functions of one variable). In some semesters, fewer than one half of the students enrolled after two weeks have completed these courses with a grade of A, B, or C. Anecdotal information suggests that many of these students expect to fail these courses on their first attempt, and hope to learn enough to achieve a respectable grade on the second try. This approach is clearly wasteful of both faculty and student time. If taking a transition course improves the first-time success rate at the higher level, then a strong case for its continuation (and even requirement for most or all mathematics majors) can be made.

Important topics such as formal logic and induction are treated rapidly at the beginning of the abstract algebra and advanced calculus courses, creating difficulties for many students who have little or no familiarity with these topics. It was felt that a more leisurely treatment of these subjects in MATH 280 would greatly improve student comprehension.

As mentioned above, many NIU students transfer from community colleges, and many of those planning to be mathematics majors come to the university with a 3 credit linear algebra course from their previous institution. In terms of rigor and abstract treatment of the material, such courses tend to be not quite at the level of the 4 credit linear algebra course offered here. A number of these students were offered credit for their previous course in linear algebra, if they would also agree to sign up for MATH 280. Approximately half of the 18 students taught during 1996 were recruited into the class in this way.

As a final consideration for offering the new course, at least two thirds of all mathematical sciences majors at NIU have an emphasis in mathematics education. The mathematical requirements for this emphasis are rigorous, falling only one course short of the requirements for those going on to graduate school (that course usually being Advanced Calculus II). Given the changing nature of instruction in mathematics in the public schools, it seems desirable to strengthen the preparation of our future teachers in the areas of logic and critical thinking. Thus, MATH 280 would appear to be an appropriate course for this special population of students.

The text selected for the initial two offerings of MATH 280 was "A transition to advanced mathematics," third edition, by Douglas Smith, Maurice Eggen, and Richard St. Andre (Brooks/Cole, 1990). This was also the text used in the course at Berkeley, and could be described as a standard choice for a transition course. Other texts which are more discursive and more informal in spirit are also available.

The class met three times per week for a fifteen-week semester, covering most parts of chapters 1-4 of the text. A rough breakdown of topics covered is as follows: formal logic (8 classes), elementary proofs (4), set theory (6), induction (6), combinatorics (3), relations (6), and functions (6). A deliberate decision was made not to directly treat the material in chapters 5-7 of the text, which is currently covered in abstract algebra and advanced calculus.

Assessment of the students was as follows: three exams were given during the semester, partly in-class and partly take-home (usually a weekend was allowed for students to work on this part). The third author found the take-home exams to be essential for measuring the ability of the students to do careful work on challenging problems, and strongly recommends this approach (the students favored it as well, even though it was more time-consuming for them). The third author did not find evidence of student collaboration (nor of appeal to higher mathematical authorities) on the take-home exams. In addition, homework was assigned, collected, and graded regularly, and points were allowed for class participation (including small group work and oral presentations at the board). There was a standard, in-class final exam (open book and open notes).

As an indication of the level of difficulty of the exam questions, here are 5 of the 10 questions posed on the third exam in Fall 1996 (entirely take-home). The second of these problems is perhaps the most challenging; four of the ten students who took the exam found the correct formula, although only one gave a complete logical justification for it.

SAMPLE TEST QUESTIONS

1. Prove that for every positive integer n, $n^3 - 3n^2 + 2n$ is divisible by 6.

2. Suppose we draw n lines in the plane (where n is a positive integer) in such a way that no two lines are parallel, and no three (or more) lines have a common point. Let r(n) be the number of regions into which the n lines split the plane. Find a formula for the value of r(n) for any n.

3. Eight students (5 male, 3 female) are to be lined up in a row for a photograph.
   a. In how many ways can this be done if the two students on the ends must be male?
   b. In how many ways can this be done if the two students in the middle must have opposite gender?

4. Let n be a positive integer, and let A be a set with n elements. Prove that A has the same number of subsets with an even number of members as it does subsets with an odd number of members.

5. Let D be the set of all people who are alive now, or who were alive in the past. Let R = {(x,y) in D × D: x is a parent of y} and let S = {(x,y) in D × D: x is a brother of y}.
   a. Find the domain and range of R.
   b. Describe the inverse relation of R.
   c. Describe the composition $R \circ R$.
   d. Describe the composition $R \circ S$.

The class average on the take-home exams tended to be in the 70's (as compared with averages in the 60's on the in-class exams). At least one student scored 90 or above on virtually all of the exams.

The homework assignments included some problems which might ordinarily be considered to fall in the category of "recreational mathematics." See example below:

A LOGIC PROBLEM

On St. Patrick's day, five lucky Irishmen in five Irish counties each caught one of the wee people and relieved him of his pot of gold. From the clues below, determine the first and surname (Conners, McDuff, O'Donald, O'Shea, or Ryan) of each Irishman, the county in which he lives, and the name (one is Angus) of the poor leprechaun he caught.

1. Kelly Ryan did not catch Paddy.
2. The Irishman from Meath, who is neither Michael nor O'Donald, caught Paddy.
3. Patrick, who lives in Wicklow, is not McDuff, who caught Thom.
4. The one who caught Willy is from Connaught.
5. Conners, who lives in Armagh, is neither Darby nor the one who caught Sean.
6. Erin is neither O'Shea nor the Irishman from Leitrim.

Interestingly, many of the students in the MATH 280 class (including some of the very weakest ones) did their best work on problems of this type. Although this problem is in some ways rather abstract, the students interpreted it as real, and provided fine examples of organized logical analysis in solving it (including a variety of graphical representations). This was a high point of the course, although the connection to future success in abstract algebra and advanced calculus is not necessarily clear.

At the end of each semester, student evaluations of the course were collected and analyzed. In addition, the third author spoke with more than half of the students in subsequent semesters about their perceptions of the course, especially as it related to their experience in the "next course." The students seemed positive about their experience, for the most part, and some felt that the course should be required for all mathematics majors.

Here is a sample of written student comments about the course, with reaction from the instructor in brackets:

- *In-class exams should have fewer problems. Take-home exams were better.*
- *Consider out-of-class groups as opposed to in-class groups.* [This can be a problem when many of the students are commuting to the campus.]
- *Discussion of one-to-one and onto functions was helpful in understanding linear algebra better.* [It is not clear to the third author whether this course should precede linear algebra, or follow it, or even be taken concurrently.]
- *Spend more time on quantifiers.*
- *I've become better educated in logic, and my problem-solving skills have improved.*
- *Helped me quite a bit with learning how to write proofs.*
- *Give more examples similar to the homework problems.* [The student desire for templates was not particularly shared by the instructor!]
- *Grading of homework assignments was too severe.* [It was certainly more severe than grading in the calculus sequence].
- *Spend more time on limits of sequences and functions.* [This certainly could be done, but would intrude directly on the ground covered in advanced calculus.]
- *Combinatorics relates to probability, but not to advanced calculus.* [Topics such as combinatorics and graph theory are attractive alternatives for a transition course, but perhaps do not tie directly to success in abstract algebra and advanced calculus.]

The student outcomes were as follows: 5 of the 18 students who took the course were awarded an A, and each of these students made an A in each of the subsequent mathematics courses which they took. None of these students really needed MATH 280; they would have succeeded in higher level courses without it, but it was enjoyable and pleasant to have them in class.

Ten of the students made a B or C in MATH 280; their grades in subsequent higher level courses were B (1), C (5), D (2), and F (2). While this record is in some respects fairly grim, it is the third author's view that many of the C grades, in particular, would have been lower without the benefit of the transition course. It is not likely that students at this ability level will show dramatic gains in comprehension in subsequent courses. However, it may be that modest gains over time can be documented, as more data from future offerings of the course become available.

One student received a D in MATH 280, and a C in the subsequent abstract algebra course. Finally, two students, both of whom were exceptionally weak, withdrew from the course during the semester. In fact, they also withdrew from the university and did not return. The third author had been advised by a colleague at a fine liberal arts college that one effect of a transition course is frequently to drive the weakest students out of the major (presumably a reaction to the perceived reality of what the study of abstract and theoretical mathematics actually involves).

In summary, the third author finds the concept of a transition course to be valid and useful in the setting of a large public university. As a cautionary note, moderate class size (10 to 15 students, say) is highly desirable, especially if small group work during class periods is to be an important feature of the experience. When the third author taught this class for the first time, he had the assistance of a graduate student, which allowed the progress of three small groups to be monitored rather carefully. However, a graduate student was not available when the course was taught for the second time, and it proved difficult for a single person to keep four working groups "on task" for the better part of a class period. There are certainly benefits to small group work of this type, but the pace of progress is very slow, and it seems that an interactive lecture format retains its validity in this setting.

The Second Tale: Trinity Christian College

Trinity Christian College is a private liberal arts college with a Christian heritage drawn from Reformed denominations. Enrollment hovers just above 600 students with roughly two-thirds of the students living in campus housing. In the 1996-97 school year, there were 15 declared math majors and almost as many pursuing a minor or a concentration in mathematics. Although in the past careers chosen by math majors centered around education, current majors have more diverse parallel interests (actuarial science, business, chemistry, music, computer science, elementary and secondary education).

At Trinity we have begun to transform various parts of our curriculum to smooth out the jumps in levels of thinking. At the freshmen level, students are now learning Calculus from a "reform" style course where reading, writing, and *thinking* are required components. Through this first year, students gradually become more and more accepting of problems that require them to explain why something is true. They also are acclimated to looking at the intricate details of a particular topic, rather than looking only at the surface algorithmic processes. By requiring this mixture of thinking, explaining, and detail analysis, the students became better thinkers and good at expressing their judgments. With the "proof-writing" stage set so effectively, the program was ready for the next level of transformation.

A related factor that pushed further change at Trinity has been the scheduling of junior/senior elective courses. Enrollment numbers allow only alternate years for each elective offered. (See table 1.) This forced junior students taking their first proof course to compete against senior students taking their fourth or fifth proof course. This created a problem. From a teaching perspective, content in logic and proof-writing had to be explained in each course, while available syllabus time was insufficient to explain the detail necessary to create good proof-writers.

Table 1: Comparison of Major Course Offerings

Old Structure		**New Structure**	
Semester 1:	Calculus I Multivariable Calculus Linear Algebra	*Semester 1:*	Calculus I Discrete Structures Linear Algebra Geometry
Semester 2:	Calculus II Differential Equations Abstract Algebra Probability & Statistics	*Semester 2:*	Calculus II Multivariable Calculus Abstract Algebra Numerical Analysis
Semester 3:	Calculus I Multivariable Calculus Discrete Structures Geometry	*Semester 3:*	Calculus I Discrete Structures Differential Equations Probability & Statistics I
Semester 4:	Calculus II Differential Equations Numerical Analysis Real Analysis	*Semester 4:*	Calculus II Multivariable Calculus Real Analysis Probability & Statistics II*

*proposed new course

At the center of this problem was the Discrete Structures course. This course contained necessary proof-writing topics, but also included content overlaps with Linear Algebra, Abstract Algebra, and Probability & Statistics. Students who had already taken several proof courses (including the above) were disinterested in the elementary discussion of these topics the Discrete course allowed; students who had not taken proof courses were overwhelmed by the pace.

A second scheduling issue at Trinity related to placement of students who had taken the AP exam and scored well. These students had three options: take Multivariable Calculus (third calculus course) in their first semester, audit/retake Calculus I in the fall, or take no math in the fall and enroll in Calculus II in the spring. Although excellent students had success with the first option, marginally good students choosing the second option developed poor work habits that followed them through their undergraduate careers. Several excellent students choosing the third option never enrolled in Calculus II and dropped mathematics as a field of study. Developing majors with poor work skills and losing others who showed potential for excellent performance as a math major were distasteful side-effects of the system.

Revision of the Discrete Structures course seemed the most natural solution. Scheduling the course in the fall semester as a sophomore course would give AP students a strong start for study of mathematics at Trinity. Teaching sophomores how to write proofs would dispel some of the current "sink or swim" mentality in the elective courses at the junior/senior level. The next issue was to determine which topics among those in a typical Discrete course would best meet the new objectives while retaining the sufficient content objectives of a standard Discrete course.

To meet objectives of the new approach, it was essential to study foundational topics such as symbolic logic and elementary set theory in detail. These topics would serve as building blocks and practice tools for proof writing. Number theory and matrix algebra were obvious topics to delete, due to course overlap, although number theory topics would reappear in applications of proof-writing and relations. Proof writing also needed heavy emphasis, so most of combinatorics and discrete probability had to be omitted. Discussion of functions, relations and a smattering of Graph Theory completed the syllabus. (See table 2.)

Table 2: Comparison of Topics between Old/New Discrete Course

Topic	*Old* Course	*New* Course
Logic	2.5 days (7.3%)	7 days (18.9%)
Set Theory	1.5 days (4.1%)	9 days (24.3%)
Number Theory	3 days (8.8%)	0 days (0.0%) topics covered in illustrative examples only
Proof Techniques, Math Induction	4 days (11.8%)	9 days (24.3%), includes conjectures
Functions & Relations	7 days (20.6%)	8 days (21.6%)
Matrix Algebra	2 days (5.9%)	0 days (0.0%)
Combinatorics	8 days (23.5%)	0 days (0.0%)
Graph Theory	6 days (17.6%)	3 days (8.1%)
ISETL	0 days (0.0%)	1 day (2.7%) plus 5 lab days to reinforce & extend current topics
Total lecture days	34 days	37 days

However, selection of these topics led to a new consideration: the service role of the Discrete Structures course. At Trinity, the Discrete Structure course is cross-listed with the Computer Science department and is a requirement of all computer science majors. Thus, loss of the overlapping content (matrix algebra, combinatorics, recursion) was a permanent loss to any student who would not be taking the additional courses. In this case, the students of greatest concern are computer science majors who are not completing a double major in mathematics. Fortunately, in our trial run in the Spring 1997 semester, there were no such students enrolled. So postponing the problem was the easy first option. But in future semesters it may be necessary to reinsert a few of the topics from combinatorics.

The next important issue was finding the best text for the course. There are several books available that seek to address the problem served by a transitional course, but not all of these books meet the content requirements for a Discrete Structures course. (See reviews of several such texts following the current paper.) The text that best seemed to meet our needs was *Conjecture and Proof, and Introduction to Mathematical Thinking* by D. Schwartz. In addition to containing the majority of the topics deemed important for the new course, the Schwartz text also has chapters on set cardinality, group theory, and analysis, and computer activities using ISETL. Missing topics (partial order relations and graph theory) were minimal and could be easily added to the course.

Because this revision of Discrete Structures was created to meet the needs of so many diverse categories of students, it is necessary to evaluate each category individually to determine the success of the course. For purposes of the current discussion we will consider five types of students: freshmen students with AP credit, math minor/math education students, students "backtracking," computer science majors with no math major/minor planned, and brilliant students. Although this classification does not comprise a partition of the enrolled students, these categories seem to be the most crucial ones to analyze.

Students who are freshmen and have AP Calculus credit. Students in this category will typically have achieved a "5" on the AP exam and will take the second course in calculus following the revised Discrete Structures course. In the preliminary run of this course there was not an exact match for this category, but there was one freshmen student who decided to pursue a major in mathematics after taking a course in Applied Calculus (business applications), so his background was a near match. This student exhibited perfectionist tendencies and was upset when he did not obtain 100% scores on homework and tests. Establishing his position in the societal/intellectual structure of the class was another area of concern. He appeared to be fearful of showing content weakness in group settings which led to an unwillingness to voice opinions about conjectures and a reluctance to collaborate in out-of-class student problem sessions. It appears that both intellectual and social maturity will have a big influence on the success of students in this category. However with early introduction of group projects (whole class and small group), the instructor can influence the development of the learning community to include these students and deflect the negative traits.

Students who are completing education degrees with concentrated study in mathematics. In this class there were three pursuing a major and one a minor in mathematics. All four were sophomores who had completed at least one calculus course with an emphasis on reading and group work. In the old "sink or swim" course offering structure at Trinity, these students would likely have been among the most frustrated students. Instead, it appeared that they had a positive response to course pace and detail. One remarkable characteristic, brought about by a lack of confidence in their mathematical instincts, was a greater reliance upon the structure (definitions and theorems) of the systems being studied. In turn, this reliance seemed to result in precision and fewer errors in proof-writing and problem solving. In the future, the Discrete course will be prior to the Multivariable Calculus course so that the mathematical maturity of future students in this category may decrease. However, prior reading emphasis in course work should provide sufficient experience for the students to retain this jump in precision.

Students who are backtracking though the system. These students have junior standing and have completed traditional calculus courses and at least one course that emphasizes proofs. Although existence of this classification is temporary due to the realignment of courses in the middle of their program of study, there were some interesting issues that may reappear with students entering Trinity in the middle of their studies. First, these students were less willing to begin at the beginning in the study of logic and proof writing. This had both good and bad consequences. While these students seemed to have a greater trust of their own mathematical instinct and were able to create more innovative solutions, these same innovative solutions were not always precisely presented and were inaccurate in detail. In the future, it may help to gently compel these students to give up poor habits early in the semester through homework interviews or class presentations of work.

Students who are pursuing a major in computer science but without a minor in mathematics. Although this classification was empty in the first run of the course, it very likely will not be empty in the future. In the past students in this category have often had poor success in prior mathematics courses and are reluctant to participate in class. To build upon their strengths, it may help to pair computer science majors with math majors and have them work together both in ISETL routine writing and in problem solving. This approach may give weaker students an opportunity to excel and to build confidence for more difficult tasks.

Brilliant students. The success of these students would occur regardless of the existence of this transition course. The one student in this classification this past semester was largely self-taught before entering college and had completed a reading-based calculus sequence before enrolling in Discrete Structures. His tendency was not to participate in supplemental group problem sessions, although he was willing to aid peers who asked him for help. For the most part he found the material easy to master on his own, so he tended to skip classes. However, he did enjoy discussions of the bigger picture that are natural consequences of the topics in the course (category theory, Russell's paradox, cardinality of infinite sets, etc.). In future semesters, it may prove helpful to provide outlets for creativity (e.g., supplemental research projects) and to increase the emphasis on reading for content mastery to keep these students engaged in the course. Since participation in the community of learners is also essential, interspersing in-class group explorations throughout the semester and collecting a report at the end of the class session may improve attendance patterns.

Of the ten students enrolled in this transition course, seven were concurrently enrolled in Probability and Statistics I. It focuses primarily on probability and probability distribution functions. Out of the seven students enrolled in both courses, two had already completed a course in linear algebra. For these two students, their proofs were similar in quality to those which had been observed in the linear algebra course. For the five students without previous experience in proofs, a noticeable improvement in the quality of proofs was observed. On the first homework assignment of the semester, students were asked to prove that the sum of deviations of a set of measurements about their mean is equal to zero. While two of these five students wrote an acceptable proof for this problem, two others provided a numerical example and considered it an acceptable proof, and the other student submitted a poorly constructed proof. On each of the two in-class exams and the final exam, students were required to write one or two proofs. The fact that four of these five students submitted acceptable proofs on the final exam provides some evidence that the quality of proofs had improved. As these students continue taking advanced mathematics courses, we will have additional opportunities to evaluate the effectiveness of the transition course.

Overall, use of Discrete Structures as a transition course seems to be successful, but long-term results will not be available for several semesters. Anecdotal evidence from students supports the positive success of the course (two students in the course decided to switch from business to mathematics for their major field of study during the semester, none dropped the major). This is supported by the formal student evaluations collected by the college. A sample of the written comments, with reaction from the instructor in brackets, follows.

- *The homework helped reinforce the stuff we learned in class. (I think ISETL was more of a hindrance than a help for the course.)* [Refining the use of ISETL is a goal of the second author, but read on . . .]

- *I liked going to the computer lab.*

- *Computer sessions were great. They helped us understand the material.*

- *Fantastic book.*

- *Book's examples often do not reflect HW.* [In the opinion of the second author, lack of template homework problems can be a good thing.]

- *It* [the course?] *could be a bit more challenging.* [An opinion not shared by all the students. On a scale of 1=very easy to 5=very difficult, the average of the course difficulty ratings by students in this course was 4.30. For comparison, the average for all Trinity courses this semester was 3.64.]

- *We need more examples like the problem sets to understand. More hands-on applications w/ the problems in class. By this I mean more group doing problems in class. Less proofs!* [Although this may be a request for template problem sets, the request for more in-class group problem sessions is valid. Reducing the quantity of proofs in the course is not an option, but reducing the number of proofs in a single problem set is an option to consider.]

In conclusion, the second author finds this experiment to be successful, although some revision and refinement of the course will likely occur in future semesters. To address needs of computer science majors, a unit on basic combinatorics which focuses largely on enumeration techniques should be included in the course. The department at Trinity also would like to investigate the addition of topics in the Christian philosophy and world-view of mathematics to the Discrete course, although the size of this content addition would necessitate adding a fourth credit hour to the course.

Conclusions and Implications

- At both Northern Illinois University and Trinity Christian College, more data are needed to further confirm the apparent effectiveness of these transition courses. These data will include more longitudinal data from those students who have already completed the transition course as well as data from additional students who will complete the transition course in upcoming semesters.

- Those who attended our session were asked to describe any transition courses which were offered at their college or university. Exactly half of the 18 institutions represented at the session offer such a course. Interestingly, all but two of these transition courses focus on discrete structures, at least in part. In addition, two of the nine institutions who do not offer a transition course noted that they use a standard course in discrete structures to fulfil this role in the curriculum. This may support the view that this content course contains numerous exercises which are easily accessible to students at this level of mathematical sophistication.

- In both courses, the motto "less is more" seems to apply. That is, rather than attempting to include more topics, many of which are usually included in a course in discrete structures, these transition courses covered fewer topics in greater depth. In particular, significant time was devoted to logic and proof, set theory, relations, and functions.

- Within the realm of pedagogy, both small groups and interactive lecture formats were used. It was noted that additional resources such as teaching assistants might help to facilitate small group interaction. One of the courses made significant use of ISETL to assist in the learning process.

- Though the nature of our institutions, the reasons for the development of the transition course, and the choice of textbooks were obviously different, the major topics of each course and many of the results of each course were quite similar. The authors are a bit surprised by the degree of convergence in the two courses, and this similarity may reinforce both the need for and the nature of transition courses.

Book Reviews

Transition Text used by paper collaborators:

Smith, D., Eggen, M., and St.Andre, R.: *A Transition to Advanced Mathematics*, Third Edition, Brooks/Cole Publishing Company, 1990. ISBN: 0-534-12234-5.

> This text (which has been used at Northern Illinois University) offers a traditional approach to a "transition" course. The core material (logic and proofs, set theory, relations, and functions) can be augmented by sections from the concluding chapters on cardinality, group theory, and real analysis as time permits. The text is generally readable, although the instructor will need to supply additional examples in the section on infinite unions and intersections of sets. Many homework sections conclude with "proofs to grade"; the third author found it most useful to specify the "proofs" which were invalid, and to require the students to explicitly indicate the flaws in them. A Fourth Edition of the text has recently been published.

Schwartz, Diane Driscoll: *Conjecture and Proof: An Introduction to Mathematical Thinking*, Saunders College Publishing, 1997. ISBN: 0-03-098338-X

> This text has the content foundations of a discrete structures course with emphasis on the transition to higher level thinking processes. The text has many helpful pedagogical tools including *Problems for Investigation* and *Computer Activities* using ISETL. The former tool the second author has found to be a good resource for classroom discussion and for group activities. The latter tool was useful for gleaning ideas for ISETL laboratory sessions; the second author repackaged most of the material into an appropriate format for class use and found a few errors in the ISETL code. The problem sets contain interesting problems but there is not an overabundance of them. The second half of the text contains supplemental topics that can be used as a final project for the course. This feature makes the text appealing to a wider audience; many will find a favorite topic in this section to include in their course. This is the text Trinity has adopted for their Discrete Structures Course.

Other available texts:

Gerstein, Larry J.: *Introduction to Mathematical Structures and Proofs,* Jones and Bartlett Publishers and Springer Verlag, 1996. ISBN: 0-7637-0203-X

> This text has essentially the same "bridge" course features as the Schwartz text but without the supplemental material for adapting the text to meet your own class needs. The level of the text appears to be slightly lower than the others. One strong feature is the chapter on Permutations and Combinations, which makes a leap into group theory from rigid motions of a polygon. The exercise sets have some interesting problems.

Mount Holyoke College: *Laboratories in Mathematical Experimentation: A Bridge to Higher Mathematics*, Springer, 1997. ISBN: 0-387-94922-4

This text is a collection of interesting and diverse topics in which students likely have little experience and through which they can develop a "feel" for mathematics and the process of investigation and discovery in mathematics. Among the sixteen lab topics are the Euclidean algorithm, polyhedra, p-adic numbers, and several involving iteration. There is a heavy computing requirement, but some programs are included in the text and others are promised in the instructor's supplements. Some chapters include either a bibliography or a suggested reading list. All chapters include exploration questions. For those who have an independent "bridge" course, this is a strong candidate for the course text.

Solow, Daniel: *The Keys to Advanced Mathematics: Recurrent Themes in Abstract Reasoning*, ISBN: 0-9644519-0-5. Order from BookMasters Distribution Center, 1444 U.S. Route 42, RD#11, Mansfield, OH 44903,

This text has two foci. One is to develop basic themes that occur in advanced undergraduate courses such as unification, generalization, abstraction, closed-form solutions, numerical methods, and axiomatic systems. This focus is at the heart of a "bridge" course. The other focus is to apply understanding of these themes in discrete mathematics, linear algebra, abstract algebra, and real analysis. The text is organized into six large expository chapters with exercises at the end of each chapter. The second author finds this too cumbersome to teach from, but others may enjoy the flexibility of assigning problems as a discovery for material not yet discussed in class. This book may best be used as a reference text for more experienced mathematicians or a self-teaching guide for beginning graduate students.

Wohlgemuth, Andrew: *Introduction to Abstract Mathematics*. To be published by Trefoil, Orono, ME. Copies may be obtained from the author. Author's e-mail address: andreww@maine.maine.edu.

This text was sent to the ACMS conference for members to evaluate. The text is written to use a "bottom-up" approach to learning the analytical skills needed for abstract mathematics. The text is divided into two sections. The first section contains twenty-three sections of "Basics" for abstract mathematics. The second has thirteen sections divided into four independent parts that offer advanced topics to supplement the basic course. A. Wohlgemuth invites interested users to send comments.

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