On Random Numbers and God's Nature *

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Abstract

I start with mathematical Platonism, an ancient stream of thought that views numbers as transcending physical reality. I join this to recent insights into mathematical randomness from theoretical computer science. Joining these streams – one ancient, one recent – yields the surprising conclusion that randomness, defined in a particular way, is part of the nature of God. I then explore some of the implications of this conclusion for our understanding of the doctrine of God's infinitude.

Computer scientists approach randomness via random numbers rather than random events. But the term "random number" is not the same as that used by experimenters for whom a random number is one generated by a process that makes any of a collection of numbers equally likely, as occurs when tossing a single die. Rather, computer scientists take a single number and ask if it is random. In the binary, "base 2," language used in computers, where "1" and "0" are the only options, all numbers are strings of 1's and 0's. Consider, for example, the numbers $.1010101010101010\ldots$ and $.01111110011110110111$. Intuitively, the latter seems more random since predicting the next digit appears impossible. (The latter number was, in fact, generated by flipping a coin, which confirms that the next digit would be unpredictable.) Computer scientists ask what "random" means for such numbers.

The first attempt to formulate a concept of randomness for sequences of numbers was by Richard Von Mises in 1919. This approach starts with the idea that for a random number written in binary, each consecutive bit should be equally likely to be a 0 or a 1. This means that as we look at increasing numbers of bits, the bits will come closer to being half 0's and half 1's – this is called the law of large numbers. However, there are strings – $.10101010101010101010\ldots$, for example – that satisfy the law of large numbers (being half 1's and half 0's) but are not random. So Von Mises focused on substrings. If one picks the substring found in positions $\{1, 3, 5, \ldots\}$ of $.10101010101010101010\ldots$ one gets the string $.111111\ldots$, which is clearly not half 0's and half 1's. This, according to Von Mises, shows that $.10101010101010101010\ldots$ is not random. He then calls any selection process that can be described by a rule like "look at every other bit" an "acceptable selection rule." He then says that a random number is one for which all substrings selected by acceptable selection rules satisfy the law of large numbers. This was a good start to defining randomness for numbers but it didn't solve the problem because it provided no way to decide which rules were acceptable.

In 1936, Alan Turing (1912-1954) defined what has today come to be known as a Turing machine. Turing's goal was to develop an abstract, unambiguous formalization of the process of evaluating a mathematical function (such as $x^2 + 2x + 1$). While not a physical machine, Turing's "machine" was a careful description of a

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step-by-step process, what we commonly call an "algorithm."[1] Turing's concept heavily influenced the development of actual computing machines that took place in the decade following. Mathematicians and computer scientists generally regard Turing's efforts as successful; the "Church-Turing thesis" named for Alonzo Church (1903-1995) and Turing asserts that any operation that can be carried out on an actual computer can (at least in principle) be carried out on a Turing machine. Thus Turing machines provide an abstract setting in which one can ask and answer theoretical questions about what computers in principle can and cannot do. Von Mises wrote before the concept of Turing machine had been defined. Its introduction made it possible to say which selection rules are acceptable, namely the ones that can be formulated as a Turing machine.

Starting with the work of Per Martin-Löf in 1966, computer scientists have formulated a detailed theory of randomness for numbers; the theory extensively uses Turing machines. It includes numerous definitions of what it means for a number to be random, for what it means for one number to be more random than another, and many other nuances. The features of this theory that are of the most interest to us here, however, are:

1. Three definitions of "random" that have been shown to be equivalent;
2. The fact that using these equivalent definitions, it can be shown that, in a mathematically precise sense, almost all numbers are random.

The definitions are intuitively appealing and can be made mathematically rigorous – and when mathematicians formulate a concept in more than one intuitively appealing way and the definitions are subsequently shown to be equivalent, it reinforces the belief that they have successfully captured a significant idea. What follows is a brief, intuitive explanation of each concept; a more technical explanation can be found in the appendix. The three concepts are:

Irreducibility

Consider two bit strings: 10101010101010101010 and 01111110011110110111. The first has an obvious pattern; the second was generated by flipping a coin 20 times. The first can be generated by this algorithm:

Repeat 10 times: output '10'

The second, however, requires an algorithm like

Output '01111110011110110111'

That is, the second string cannot be reduced to one shorter and simpler than itself. The underlying intuition is that a string of n bits is random if any algorithm able to generate it requires at least n bits, i.e., the string is irreducible. Infinite strings are random if they cannot be reduced to finite expression.

Martin-Löf randomness

A string like 10101010101010101010 has an obvious pattern; a string like 01111110011110110111 does not. Of course, a string could look like it has a pattern near its beginning but then become patternless. Martin-Löf randomness captures the idea that a string which is not random has a finite pattern and maintains that pattern throughout a possibly infinite length. A random string is one that lacks a pattern.

Constructive martingales

Suppose a string of bits is revealed one bit at a time. The intuitive idea behind the martingale concept of randomness is that there is no betting strategy that would enable one to profit by predicting the next bit. That is, randomness defined in this way corresponds in a meaningful way with unpredictability.

[1] A precise definition can be found on the internet or in any introductory text on computing theory.

So, the underlying intuition behind these three concepts of random number is that a number is random if it is irreducibly infinite, has no finite pattern, and is unpredictable. The key point for us in this paper, however, is that random numbers are numbers. Thus:

> *If numbers have indeed existed in the mind of God from eternity, randomness is and always has been part of God's nature.*

I will explore the theological implications of this idea in the next section and the scientific implications in the section following that.

Theological implications

What does the idea that randomness is part of God's nature tell us about God?

First, let's consider what it does *not* tell us. The popular concept cited above is that randomness means not having a governing design, method, or purpose; without order; without cause. This popular concept is what makes the idea that randomness might be part of the divine nature seem strange or shocking. But algorithmic randomness is quite different from the popular concept of randomness and *is* informative about God's nature – even under a Christian theology which has always affirmed that God has designed the world, acts with method and purpose, and is orderly. Unlike popular notions of randomness, under the mathematical hypothesis of numbers being ideas in God's mind, the properties of random numbers are necessary properties of God's nature[2] and can enrich our understanding of God's infinitude. Before we can see what these properties add, though, we need to see how systematic theologians have historically understood divine infinitude. Here is a typical list of divine attributes [3, pp. 36-37] to provide a context for the analysis that follows:

The Nature of God

Divine sufficiency (primary and essential attributes of God inapplicable to creatures and not communicable to creatures)

- Uncreated
- Unity
- Infinity

The divine majesty (relational attributes of God displaying God's way of being present, knowing and influencing the world)

- Omnipresence
- Omniscience
- Omnipotence

[2]Note that this analysis does not simply give analogies taken from nature; rather it provides propositions that are necessary truths about God, subject to the limits of human language.

The Character of God

The divine thou (active and interpersonal attributes belonging to the divine-human relationship and analogous to personal experience);

- Incomparably personal
- Spiritual
- Free

The divine goodness (moral qualities intrinsic to the divine character)

- Holiness
- Goodness
- Compassion

In this taxonomy, infinity is one aspect of God's sufficiency; however, Christian thinkers' understanding of God's infinitude has varied over time and across religious traditions. The notion that God is infinite seems to have first appeared in Christian writings among early Gnostics. Augustine wrote that God is infinite in wisdom and is unbounded not in the sense of being suffused throughout space, but rather God is infinite "in another way" although he did not comment on what this way is [1]. Thomas Aquinas devoted Question Seven of the first part of his *Summa Theologica* to God's infinitude. He conceives of it as meaning that God is not limited in any way and is infinite in perfection in the sense that God's perfection cannot be diminished or increased. He wrote that God is unique in being infinite, that no bodily thing can be infinite, and there cannot be an actually infinite number. This latter notion originated in Aristotle's idea of "potential infinity" – for example, integers increasing without bound but not reaching a limit.[3] Some medieval scholars distinguished extrinsic and intrinsic infinity. The concept of "extrinsic infinity" was based on the integers continuing without limit; "intrinsic infinity" was based on the notion of a finite space being infinitely divisible. They suggested that God's infinitude is intrinsic not extrinsic; this seems to be a way to affirm God's infinitude while avoiding the notion that God is infinite in extent – but this was ambiguous about the relationship of intrinsic infinitude to God's nature. John Calvin's concept of God's infinitude was "beyond our senses." Many theologians have pointed out that God's infinitude is not separable from other attributes – it is part of what it means for God to be omniscient, omnipresent, and omnipotent. Some pointed out that God's infinitude, when applied to time, is God's eternality; when applied to space, is God's omnipresence. Herman Bavinck emphasized that God's infinitude applies to character attributes as well as sufficiency and majesty and in this way is quite different from a quantitative notion of infinity [2, pp. 159-160]. The principal common theme, however, that runs through these notions is "without limit." And the etymological basis of infinity is 'unlimited.'

I can see two ways that the idea of divine randomness can enrich our understanding of God's infinitude: (1) it can serve a pedagogical role by providing images that enable us to form clearer concepts of divine infinitude, thereby enriching our worship and (2) it can introduce aspects of divine infinitude that had not been previously noted.

[3]Following the work of Georg Cantor, mathematicians today would say that Aquinas was incorrect. Not only are there actually infinite numbers, there are infinitely many of them of infinitely many different sizes. For Aristotle, numbers were quantitative aspects of physical things. Aquinas seems to have used this Aristotelian concept in saying there cannot be an actually infinite number, although he does not explicitly mention Aristotle when he says this. This is one place where Christian Platonism enjoys a decided advantage over Aristotelianism. Seeing numbers as ideas in God's mind removes the conflict Aquinas saw between God being uniquely infinite and there being actually infinite numbers – actually infinite numbers can exist because they participate in the divine infinitude. For a discussion of Cantor's work and its theological implications, see [4].

(1) The integers provide an image that many theologians have used to illustrate God's infinitude. The concepts of randomness discussed here follow in that tradition – an infinite number that is irreducibly random is one that cannot be described by repetition of a finite string; a Martin-Löf random number lacks a pattern that can be generated by any (necessarily finite) algorithm. Both of these concepts provide images of the idea that God cannot be described in terms of any finite thing – no complete description of God is possible.

(2) The martingale definition of randomness introduces an aspect of the divine nature that I have not seen discussed in connection with the doctrine of divine infinitude, namely the element of surprise or mystery. Random numbers are mysterious, such that no matter how many bits of one have been revealed, the next bit is still unpredictable. Saying that such numbers exist in the mind of God provides an image of the idea that one may indeed understand aspects of God truly and may learn more, but never come to the point where there are not further aspects of God that are surprising. Put differently, no matter how much knowledge one has of God, God's mystery remains unfathomable.[4]

In summary, discussions of divine infinitude that are informed primarily by the image of the integers lead one to the idea that God is unlimited, but not much more. A comprehension of divine randomness extends this understanding, nuances it, and enriches it.

References

[1] S. Augustine. *Confessions*. Grand Rapids, MI: Christian Classics Ethereal Library, Book VII, Chapter XIV.

[2] H. Bavinck. *Reformed Dogmatics, Volume Two, God and Creation*. Baker Academic, 2004.

[3] T. C. Oden. *Classic Christianity, A Systematic Theology*. HarperOne, 1992.

[4] C. Tapp. Infinity in Mathematics and Theology. *Theology and Science*, 9(1), February 2011.

[4] When presenting this concept at a recent conference, one conferee commented, "My tradition has always focused on God's covenantal faithfulness, but you are asking us to see God very differently." I found it quite heartening that the analysis of divine randomness opened a new understanding of God for this person.