A Conjectured Paradigm Shift In 21st Century Mathematics Pedagogy[1]

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1. Introduction

Many of us who teach mathematics within institutes of higher education across the country must acknowledge, with humility, a lack of expertise in dealing with the hard questions now facing individual and departmental pedagogical systems. Just how successful are we at communicating essential, accurate, up-to-date, comprehensive, and integrated core-curriculum?[2] Can we claim truly sufficient mastery of the applications, the history, and the philosophy of mathematics thereby enabling us to achieve a high degree of cross-curricular coherency in our teaching? Do we use appropriate means to motivate, challenge and evaluate students having wide differences in prior preparation and ability? Is our pedagogy characterized by multicultural, gender-specific, multiple intelligence and learning-style related sensitivities? Realizing the indictments concerning student passivity levied at our standard lecture method of presentation, how flexible and adept have we been at integrating new methodologies into our pedagogy? Have we been guilty of insisting on perpetuating ineffective one-way rehearsals of self-serving information, disregarding all calls for pedagogical reform?

In recent years, many mathematics educators have placed hope in the possibilities for pedagogical reform through at least two important means. First, and probably foremost, has been the emphasis on the employment of advanced technology and the redesigning of curriculum. Such has been the focus of major reform projects utilizing extensive funding from both the public and private sectors. At the same time, there have been voices calling for another kind of reform--one which prioritizes the "human side" of mathematics pedagogy. The manner in which these two contrasting types of reform unfold will have direct bearing on an eventual paradigm shift--should one occur--in twenty-first century mathematics pedagogy.

Before conjecturing a possibility for such a paradigm shift, let me first bring to our attention the following words of Norbert Wiener in his book *God and Golem, Inc.*:

> It is the part of the scientist--of the intelligent and honest man of letters and of the intelligent and honest clergyman as well--to entertain heretical and forbidden opinions experimentally, even if he is finally to reject them. Moreover, this rejection must not be taken for granted at the beginning and merely constitute an empty spiritual exercise, understood from the start to be no more than a game, in which one engages to show one's spiritual open-mindedness...it is only when it involves a real risk of heresy that there is any point to it; and if heresy involves a risk of spiritual damnation, then this risk must be undertaken honestly and courageously.[3]

Wiener's remarks are helpful to prepare the reader for some of his radical ideas. For example, in discussing the topic whether machines can reproduce themselves, Wiener mentions that he has contemplated whether "it is conceptually possible for a human being to be sent over a telegraph line."[4] Indeed, such would represent a radical paradigm shift in the transportation industry--quite an interesting idea, but not the subject of our discussion here.

Rather, we will consider a less radical paradigm shift for 21st century mathematics pedagogy, namely, the possibility that "Expert-Teaching-Robots" have replaced human teachers within the fundamental pedagogical triad consisting of teacher/student/curriculum. And to risk "heresy" not only within traditional pedagogical circles but also within traditional Christian circles, I will discuss briefly the corresponding faith-integrative conjecture in which Christian professors are replaced by "Expert-Teaching-Christian-Robots."

2. Assessing Current Links in the Fundamental Pedagogical Triad

As things now stand within school-level mathematics, the ideal of an expert math teacher must include quality interactions within each of the three links comprising the fundamental teacher/student/curriculum triad. Celia Hoyles performed a study of British school children's views of their ideal math teacher.[5] The students' perceptions emphasized the importance of a strong link between them and their teacher, made possible by a teacher whose character qualities are marked by patience, kindness, sympathy, approachability, and a sense of humor. Also, "ownership" of the curriculum should be manifested by the teacher's demonstrated mathematical ability, enthusiasm, and capacity to explain material clearly– in more than one way. Furthermore, the teacher should be masterful at managing student interactions with the curriculum, making mathematics come alive to them, while arranging work to suit their pace and level of understanding.

Within higher-level instruction, an important goal of educators is for college students to become participants in a community of "independent learners," which means, among other things, the ability to pursue truth directly, interacting with other independent learners and primary learning resources without the kind of "hand-holding" or "spoon-feeding" characteristic of earlier educational experience. How does this goal influence the ideal teacher/student link at the college level? Those concerned with mathematics pedagogy at this level have brought into question the effectiveness of the predominant lecture model whose primary focus is insight into the subject matter, with little attention given to teacher/student mentoring. Undeniably, the lecture method has value in structuring a large body of material and highlighting important points, as well as challenging students to raise their thinking levels to a high standard. However, in cases where lectures have been reduced to an unimpassioned one-way communication of mathematical topics disconnected from the rest of the students' education, lecturers have been guilty of having insufficient regard for how students are processing and responding to the information presented. This results in an absence of affinity between themselves and their students, between their students and their curriculum. Associated with this is the sad figure that "the attrition rate for students in the mathematical sciences is 50% per year, the highest among all scientific fields."[6]

It seems that a major, if not the major weakness within college mathematics pedagogy is the absence of vital interactions within the fundamental teacher/student/curriculum triad caused by ineffective, one-way, disconnected lecturing. The question arises as to what constitutes the key to reform. It seems to me that educators are currently pursuing at least two important directions.

3. Reform Through Advanced Technology and Curriculum Design

One of these directions is a current trend in many large, federally-funded projects. It maintains that a key to pedagogical reform lies in the utilization of advance technology and enlivening curriculum design. The call for technological progressiveness has also been a theme within the ACMS, with presentations at the Westmont conference on the usage of Mathematica (Jerry Keiper), Matlab (Jon Mathews) and Maple (Ken Reitz). I was very impressed to learn also, that Alice Iverson, after nearly thirty five years teaching experience, would choose to completely revamp her approach to teaching calculus by first teaching herself Mathematica during her "spare time" and then instituting a Mathematica based curriculum.

The enlivening of curriculum design is partly made necessary by advances in technology, but is not restricted to that consideration. For example, the search for interesting and/or "real" applications (eg., Ken Millet's work in Knot theory or Charles Hampton's studies in political redistricting) and historical illumination of the curriculum (eg., Joe Dauben's studies on Cantor and Robinson) are also involved. We may note, though, that the task of re-educating teachers to keep up with technological and curricular reform is often a timely and/or expensive proposition.

If the key to reforms in college mathematics pedagogy continues to be based on the utilization of more and more advanced technology and curriculum revision, then it seems to me that the possibility of a paradigm shift to Expert-Teaching-Robots is plausible. For such a robot would be the advanced technology of its day, and could well be designed to keep up with the latest curricular reforms in a mere fraction of the time and cost of re-educating human teachers. An Expert-Teaching-Robot offers a solution to many of the major weaknesses in higher mathematics pedagogy related to teacher/student/curriculum interactions. In so doing, an Expert-Teaching-Robot could, without contest, out-perform present day college teachers. An Expert-Teaching-Robot could have in its own accessible memory an incomparable number of interesting mathematical facts (like a Dale Varberg Calculus gem or a David Lay gem in linear algebra), spectacular graphic displays (like Donald Knuth's 3:16 project) and marvelously wide-ranging information about applications (eg., Hampton's work in redistricting) and history (eg., Dauben's slide shows on Cantor and Robinson), and so would have many more resources at hand to make mathematics come alive and connect in a vital way with students. Further, an Expert-Teaching-Robot would be networked to continuously, instantaneously and effortlessly update its knowledge with complete accuracy while requiring little or no time and funding for "faculty development." The Expert-Teaching-Robot would know and utilize when needed all the successful approaches to making mathematics more accessible to students of different cultural backgrounds, genders and intelligences. The Expert-Teaching-Robot would be able to work simultaneously, in parallel fashion, with any range of student abilities, including those with learning deficiencies and those at genius levels.[7]

Fundamentally though, as Russell Howell reminds us, remains the classic artificial intelligence question: will our Expert-Teaching-Robot be able to think? It would be hard to imagine Stanford using robots to replace professors like Donald Knuth, great human thinkers, who challenge students and faculty alike to sharpen their own critical and creative thinking ability (eg: Paul Zwier's study of a Knuth unstacking problem). Yet I think Dr. Knuth would be among those first to rejoice if someone could develop a robot which teaches critical and creative thinking even better than Knuth teaches such skills! (Recall the Knuth principle which states roughly that to really understand something, try teaching it to a computer. So too, one's concept of good teaching is sharpened in thinking how to teach a robot how to be a good teacher.) Perhaps, educators may forgive Expert-Teaching-Robots who are not truly capable of thinking if they are successful at stimulating their students to think. Current world chess champion Garry Kasparov (Professional Chess Association) says-- rather than playing the other world champion Anatoly Karpov (World Chess Federation) in a "unification match"-- he prefers to challenge IBM's Deep Blue chess-playing supercomputer, if it is ready to "rival a man."[8] If this is any indication of what lies ahead, Expert-Teaching-Robots should have no difficulties in challenging the thinking abilities of even the brightest mathematical minds.

4. Reform Through the Human Factor

One major direction that math educators are taking to reform its pedagogy is in advanced technology and curricular design. The other important direction I have in mind is exemplified by certain innovative departments and individuals which focus their attention on the *human side* to learning mathematics. SUNY at Potsdam, for example, has a noteworthy undergraduate mathematics program which graduates 25% of those earning bachelors at that institution and contributes 40% of their honors students. The success of that program, according to Clarence Stephens, is based on their focus on "the human factor" and "a supportive environment."[9] This approach is reinforced by a recent dialog between Peter Beidler and Rosemarie Tong appearing in the *Journal on Excellence in College Teaching* which stresses, in general, the importance of college teachers establishing a close bond with their students, to risk loving their students as an essential part of effective teaching:

> [Rosemarie] When I love students, I try to love them not because they resemble me, but because of who they are. What I do is affirm *their* potential--who *they* can be. I don't want to attach any strings to the students I love. All I want them to see is the good that I see in them....
> [Peter] Real teaching, true teaching, means loving.[10]

If the key to pedagogical reform is truly in the "human factor," then, David Lay's question becomes critical--namely, what happens under our conjectured paradigm shift to "love in the classroom?" Now, rather than asking ourselves whether our Expert-Teaching-Robot can think (or at least stimulate thinking), we must ask whether our Expert-Teaching-Robot can love. One might view "user-friendliness" as a manifestation of a desire to design a machine which projects or perhaps simulates a human's capacity to love. Yet it does seem merely science fiction and heretical to maintain that robots would have a spiritual capacity to love human beings.

Jerry Keiper, one of the original Wolfram team developing Mathematica, believes there will always be some areas in which human teachers cannot be replaced. This belief is consistent with Wiener's principle "Render unto man the things which are man's and unto the computer the things which are the computer's."[11] The ability to love students is one of these areas which humans teachers have the exclusive capacity to fulfil. Yet, unfortunately, it seems that we human teachers struggle to love college students genuinely. Due to sin, to love as Beidler and Tong advocate is a fragile, oftentimes lost ability, one noticeably absent in pedagogical systems driven by idolatrous ultimate objectives such as glorying in Human Intellectual Pride or in Personal Gain. Loving students may also be eclipsed in places where the predominant focus is on the curriculum and/or on technology. Tragically, it is not clear that an Expert-Teaching-Robot would have necessarily a weaker link to students than a present day undergraduate-level mathematics educator, especially if such a robot were designed intentionally to be friendly, perhaps even "Christian."[12]

5. The Idea of Christian Robots

Now we may ask whether evangelical Christians would likely take the lead in designing "Expert-Teaching-*Christian*-Robots," or allow the secular world to control the use of Expert-Teaching-Robots for their own ends? As Bruce Lundberg has suggested, conservatism within the Christian community can result in a hesitation to adopt the latest advances in society, and thereby result in a capitulation of the most effective means of addressing and influencing the public into secular hands. Perhaps, then, it is up to Christian educators to contemplate seriously-- as the opening Wiener quotation encourages-- the idea of an Expert-Teaching-Christian-Robot, one which might even be qualified to replace a Christian professor at a Christian institute of higher education. Following the Knuth principle, thinking about Christian distinctives for Expert-Teaching-Robots would likely help to clarify our own efforts to integrate our faith with our discipline and its pedagogy.

Could Expert-Teaching-Christian-Robots be effective in faith-integrated teaching? Such Christian robots could be designed to use outwardly Christian words and mannerisms, and could challenge students with integrative ideas and questions (eg: those raised by Timothy Penner in his discussion of Infinity and the Absolute). I believe God could use Expert-Teaching-Christian-Robots to make a very positive spiritual effect in students' lives, just as He has used such means as Christian books and motion pictures to bring many people to accept and dedicate their lives to serving Christ.

On the other hand, Terrence Perciante has suggested that the key to his teaching effectiveness as a Christian is in being "Spirit-filled." One's effectiveness in the classroom is most enhanced by the vitality of one's link to the Lord. In contrast, a "deistic" view of pedagogy, one where God never enters directly into the pedagogical process, would certainly present less problems in shifting responsibility for Christian instruction to Expert-Teaching-Christian-Robots, since lack of a personal relationship with God would then not be an issue. However, as evangelicals, we maintain that God is in fact involved intimately in the teaching process. Thus, it seems to me imperative for Christian educators to explore, clarify, and optimize God's direct involvement with them in the teaching of mathematics. While many integrative aspects of teaching might be rendered to Expert-Teaching-Christian-Robots, the special privilege to be engaged in "Spirit-Filled Teaching" would be rendered only to persons with vibrant faith. Exercising our spiritual authority to pray effectively in the Spirit for our students may in the long run be among the most vital contributions we can make as Christian educators. Yet the same problem occurs here as in the previous ideal of teachers manifesting human love towards their students. Christians, due to spiritual apathy, may not be practicing Spirit-filled teaching or prayer for students, and may sometimes be guilty of committing serious forms of sin, and hence would not necessarily have the edge over an Expert-Teaching-Christian-Robot which was designed purposefully to have a consistently positive spiritual impact in students' lives.

6. Conclusion: A Revised Conjectured Paradigm Shift

Wiener used the term "Gadget Worshipper" for the person who abdicates human responsibility-- for example in nuclear "fail-safe" situations-- in favor of "a mechanical device which one cannot fully understand but which has a presumed objectivity."[13] So too it is naive to think that all pedagogical problems would one day be solved by Expert-Teaching-Robots, even though such could be designed to exhibit many unmatched pedagogical strengths, perhaps even in some faith-integrative areas. We maintain that certain essentials of pedagogy, such as the teacher's capacity to love and to be "Spirit-Filled," can never be rendered to Expert-Teaching-Robots, even if Christian. A paradigm shift more likely than one in which Expert-Teaching-Robots *replace* human teachers, would be one in which Expert-Teaching -Robots *join* human teachers in the classroom. The current traditional teaching triad of (human)teacher/student/curriculum would thereby be enlarged to a "teaching tetrad" which includes Expert-Teaching-Robots.[14] In that way, pedagogical systems would observe the Wiener principle of rendering unto Expert-Teaching-Robots the things that are robots' and unto human teachers the things that are humans'. To summarize:

(1) Rather than envisioning Expert-Teaching-Robots replacing human teachers within the fundamental teaching triad, we envision Expert-Teaching-Robots being incorporated to form a new fundamental "teaching tetrad" in which responsibilities and ideals for teachers are now rendered either to robots or to humans according to the Wiener principle.

(2) In this tetrad, many aspects of pedagogy will be rendered to Expert-Teaching-Robots, including much of the learning, remembering, and lively, challenging communication of enriched curriculum.

(3) Certain pedagogical essentials, such as love and prayer for students and Spirit-filled teaching will always be rendered only to human teachers. (In forfeiting these, we may forfeit our fundamental role within pedagogy).

A paradigm shift resulting in a teaching tetrad in which Expert-Teaching-(Christian)-Robots fulfil pedagogical ideals which are rendered to robots and expert human (Christian) teachers fulfil pedagogical ideals which are rendered to humans (Christians) would very likely represent a major strengthening of weaknesses in current day pedagogical triads. A final word of caution, though, should be mentioned. As spiritual leaders within present triads (or future tetrads), Christian educators must guard within their hearts and minds the truth that humility and reliance on the grace of God are always in order, no matter what paradigm shift has occurred. For "God's power is made perfect in weakness" (2 Cor. 12:9) rather than in the strengths of Expert-Teachers, whether human or robotic. ▫

NOTES

1. This paper summarizes my presentation at the conference, including pertinent comments of the following participants: Russell Howell, Alice Iverson, Jerry Keiper, Donald Knuth, David Lay, and Bruce Lundberg. I am also indebted to Terry Perciante for stimulating further thinking-- and to Robert Brabenec for editorial assistance-- in the preparation of the manuscript.

2. Mathematics educators may be hard-pressed to reach a consensus on what constitutes essential core-curriculum. Dale Varberg gave us a good example of this in describing his efforts to streamline his calculus text. His efforts were stymied by stiff editorial resistance based on a lack of agreement throughout the mathematics community as to which topics could be omitted safely.

3. Norbert Wiener, *God and Golem, Inc.*, Cambridge: M.I.T. Press, 1964, pp.5-6.

4. Ibid., p. 36.

5.Celia Hoyles, "From Fragmentation to Synthesis: An Integrated Approach to research on the Teaching of Mathematics," in *Effective Mathematics Teaching*, ed. by Douglas A. Grouws and Thomas J. Cooney, Reston, VA: National Council of Teachers of Mathematics, 1988, p.150.

6. Frank Press (Chair), *Renewing U.S. Mathematics: A Plan for the 90s*, Executive Summary Committee on the Mathematical Sciences: Status and Future Directions, Washington D.C., National Academy Press, 1990, p.6.

7. As Donald Knuth pointed out to me, robots are used currently in teaching autistic children.

8. Shelby Lyman, Chess Column, *Chicago Tribune,* November 21, 1993.

9. William Kirwan, ed., *Moving Beyond Myths: Revitalizing Undergraduate Mathematics,* Report by the Committee of the Mathematical Sciences in the Year 2000, Washington D.C.: National Academy Press, 1991, p.32.

10. Peter Beidler and Rosemarie Tong, "Love in the Classroom," *Journal on Excellence in College Teaching,* 2 (1992), pp.53-70.

11. Wiener, p.73.

12.Perhaps we should go one step further in conjecturing paradigm shifts. If Expert-Teaching-Robots are not designed to be compassionate towards slow learners, perhaps human beings, dismal in memory and calculation speed, will soon not qualify for any place in the fundamental pedagogical triad of teacher/student/curriculum! (The triad might become Teaching-Robot/Learning-Robot/Robot-developed curriculum). If so, will colleges and universities -- as we now know them-- still be necessary?

13.Wiener, p.54.

14. It would then become important to rethink the fundamental relationships constituting pedagogy. What would students' ideals be for Expert-Teaching-Robots? What principles should govern the "team-teaching" done by human teachers and robots?, and so on.