The Soviet Concept of the Correlation of Forces

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Introduction

One of the most frightening features of the Cold War between the United States and the Soviet Union was the enormous number of nuclear weapons each side acquired. Table 1 lists the total number of stockpiled weapons belonging to both countries at five-year intervals from 1945 to 1995. Figure 1 presents the same data graphically[1].

Year	US	USSR
1945	6	0
1950	369	5
1955	3057	200
1960	20434	1605
1965	31265	6129
1970	25742	11643
1975	26675	19443
1980	23387	30062
1985	22941	39197
1990	20684	33515
1995	11226	15615

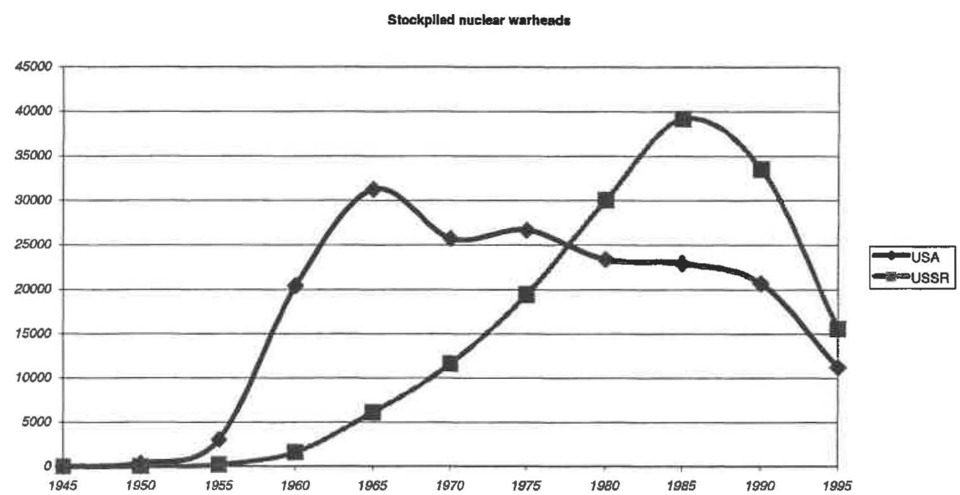

Table 1 – Stockpiled nuclear weapons

What led these two countries to acquire so many weapons? American thinking can be explained fairly briefly. The United States developed nuclear weapons first and was the world's only nuclear state in the years immediately following World War II. Nuclear weapons had ended that war and were widely viewed as having saved enormous numbers of American lives by eliminating the need for a US invasion of Japan. Thus the nuclear arms race began with a belief on the US's part that nuclear weapons would make the US safer by preventing further wars. However, as hostility grew between the US and the Soviet Union in the late 40s and early 50s and as the

USSR developed nuclear weapons of its own, the US increasingly thought in terms of a possible military conflict with the Soviet Union. US decisions about numbers of weapons were based on "targeting doctrine," a systematic plan for selecting militarily significant targets in the Soviet Union for attack in case of a war. The fact that the US aimed at military and not civilian targets largely accounts for the rapid increase in numbers of weapons between the mid-50s and the mid-60s – there is always another airfield from which military planes could be launched or another factory that could support a war effort.

By the late 50s, however, another factor entered the calculation. Based on ideas drawn in part from game theory, a branch of mathematics, Hermann Kahn and others shaped the notion of deterrence[2]. By the end of Robert McNamara's tenure of Secretary of Defense (1961-1968), deterrence, in the form known as MAD or mutually assured destruction, was well-established as the conceptual basis for US security vis-à-vis the Soviet Union. That is, from the perspective of deterrence, parity between the US and the USSR and a secure second-strike capability for each side were seen as desirable since they would characterize a situation in which neither side would have an incentive to attack the other. Thus, since parity was desirable, there was no further reason at this time for the US to continue to increase its nuclear stockpile; in fact, the US stockpile declined in the late 60s.

The Soviet Union, however, engaged in a very different analysis, one that was far more sophisticated mathematically. In order to understand thoroughly what the Soviets were doing and why, we need to take a brief historical excursion.

Historical background

The basis for Soviet Cold War thinking can be traced at least as far as ancient Greece. Aristotle saw human beings as possessing an intuition capable of discerning valid first principles about the world. He never explained where such an intuition might originate, but nevertheless, he held that it existed. Thus, from this perspective, deductive reasoning, such as is used in Euclid's *Elements*, yields absolutely certain truths about the world. Aristotle did not believe that the natural world was fundamentally mathematical in its structure. However, by the seventeenth century, belief in the scope of mathematical thinking had vastly expanded. For example, in 1623, Galileo wrote:

> *Philosophy is written in this grand book, the universe, which continually stands open to our gaze. But the book cannot be understood unless one learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth.*[3]

A generation later, Descartes envisioned a mechanical universe susceptible to mathematical analysis and proposed a method for analyzing it. Following him, Liebniz envisioned a rational, universal calculus that could be used to formulate and solve problems in all areas of human activity. In 1677, he wrote:

> *All inquiries that depend on reason would be performed...by a kind of calculus...And if someone would doubt what I advanced I should say to him: Let us count, sir; and thus by taking pen and ink, we should soon settle the question.*[4]

The foundations were being laid for a view of the world that saw it as rational, mechanical, and capable of being described by mathematics. But the main event that advanced this perspective was Isaac Newton's discovery of the laws of gravitation. His P*rincipia de Mathematica* was published in 1687. In the *Principia*, Newton showed how mathematical ideas that could be very simply and briefly expressed (such as the inverse square law for gravitation) can account for planetary motion. That is, from the point of view of his contemporaries, Newton's mathematics had solved a problem that had puzzled the best minds for millennia, that of human beings' place in the universe. This work captured the imagination of his generation. By the mid-eighteenth century, "Enlightenment" scholars were convinced that all science and technical arts could be pursued by the same rational method. They promoted this view in their grand publication project the *Encyclopedie*. Many thinkers of this era were optimistic that comparable laws could be found for all dimensions of the natural world, including human society. That is, it would be possible to develop a rational science of society and a rational social order that would avoid war and greatly advance human well being.

Starting in the reign of Catherine the Great (1762-1796), these ideas began to become popular among the Russian intelligentsia. By the late nineteenth century, they dominated intellectual life. Karl Marx' work on economics in the mid-nineteenth century was also built on this same foundation of belief in a rational, mechanical social order. He saw societies as embodying the law of class conflict and evolving toward an ultimate goal of a classless society. When he died in 1883, his most prominent disciple, Friedrich Engels began his eulogy of Marx with the words, "Just as Darwin discovered the law of development of organic nature, so Marx discovered the law of development of human history." Such ideas were widely circulated in late nineteenth and early twentieth century Russia and provided a fertile climate for the Communist Revolution of 1917, led by Vladimir Lenin.

Soviet military thought

Soviet society was a planned, centrally managed society based on the ideal of rational, scientific thought and the Marxist notion of the classless society. Military thought was similarly expected to be "scientifically based." Comments to this effect are frequent in Soviet military literature. This requirement is derived from Marxist-Leninist principles of scientific determinism that essentially declare that all phenomena are governed by laws and that no phenomena occur outside of laws. Thus, if the laws that govern a situation can be fully understood and the situation itself fully described, the behaviors that will occur are predictable. Thu Soviet scholars invested a great deal of effort into attempts to understand these laws and decision-makers aimed to base their decisions on them as well.

The Soviets believed they knew the laws of war. These laws were expressed by the Lanchester equations, first articulated by an English mathematician, F. W. Lanchester, in 1914. They were also discovered independently by a Russian mathematician, N. Osipov, in 1915. In their simplest form, they can be written:

$$\frac{dx}{dt} = -ay$$

$$\frac{dy}{dt} = -bx$$

x and y denote the size of two opposing forces and a and b denote attrition rates occurring in battle. The ratio b/a is called the **correlation of forces**. This form of the equations assumes that each side's forces are completely homogeneous; that is, they are not made up of different components having differing capabilities and vulnerabilities. There is also a heterogeneous form of the Lanchester equations:

$$\frac{dx_i}{dt} = -\sum_{j=1}^{n} \beta_{ij} y_j, i = 1,2,\ldots,m$$

$$\frac{dy_j}{dt} = -\sum_{i=1}^{m} \gamma_{ij} x_i, j = 1,2,\ldots,n$$

In this form the x_i's and y_j's denote the size of each component of each side's forces and β_{ij} (for instance) represents the attrition rate of x_i due to y_j. Expressions for the COF for this system of equations were developed as well. Note that "homogeneous" and "heterogeneous" are being used here to denote the structure of each side's forces not the form of the equation as is typically done in the theory of ordinary differential equations. Note also that these are deterministic equations; there are also stochastic versions.

The Lanchester equations have been widely studied and are still the object of current research. However, for our purposes here, the simple form shown above will suffice. These equations are easily solved if we eliminate the parameter t by dividing the second equation by the first:

$$\frac{dy}{dx} = \frac{bx}{ay}$$

$$a(y^2 - y_0^2) = b(x^2 - x_0^2)$$

Consider an example. Suppose two opponents begin with equal size forces. For simplicity let's set this common value to 100. If the two sides engage in battle, we can graph their attrition of their forces. See Figure 2.

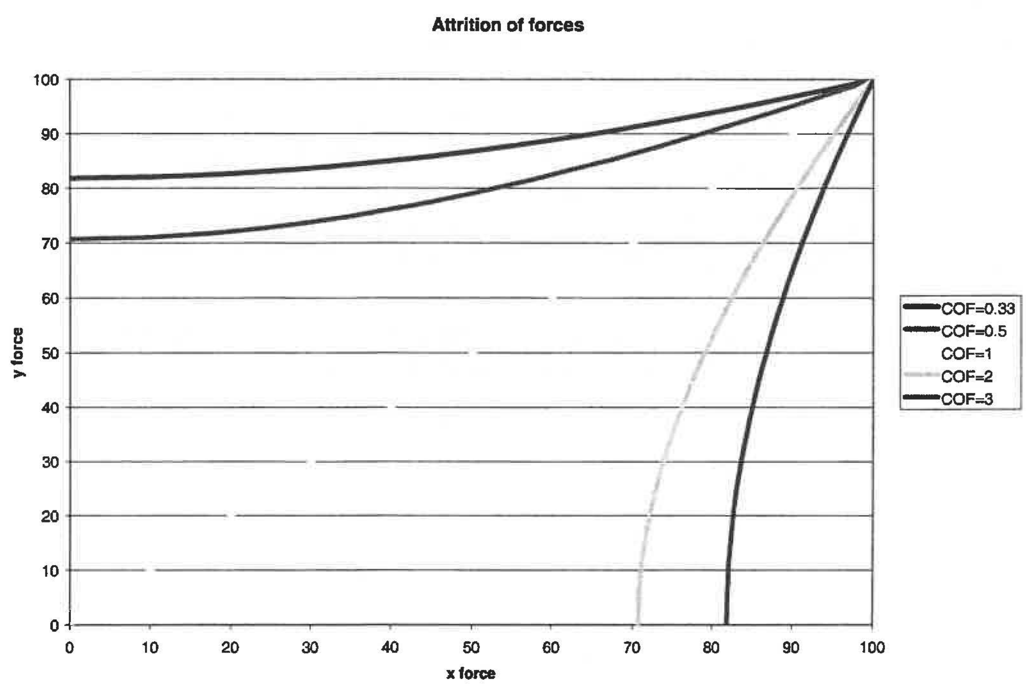

Figure 2 – Attrition of forces as determined by the Lanchester Equations

Note that a favorable correlation of forces gives a side an enormous advantage. For example, if COF = 0.5, the y forces can annihilate the x forces with less than a 30% loss. IF COF = 0.33, they can annihilate the x forces with a loss of less than 20%. In practice, adversaries rarely fight until one side is annihilated. Suppose the y side has the advantage and the x side surrenders when it loses 15% of its force. If COF = 0.5, the y side still retains 92.8% of its force; if COF = 0.33, it retains 95.3%.

Uses of the correlation of forces

Because the Lanchester equations were seen as the laws of war, the concept of the correlation of forces obtained enormous credibility as a measure of the effectiveness of the Soviet forces. Many formulae were developed for computing it in various situations; some even attempted to include intangible factors such as *esprit de corps* and strategy. When weapons and equipment as well as personnel were incorporated in its computation, it was called the correlation of forces and means (COFM). The 1979 version of the Soviet Military Encyclopedia gave a non-technical definition of the COFM that shows how much the Soviets regarded it as a factor in a natural law:

> *[The COFM is] an objective indicator of the fighting power of opposing sides, which permits a determination of the degree of superiority of one of them over the other. The correlation of forces and means is determined by a comparison of existing data on the quantitative and qualitative descriptions of subunits, units, combined units and armaments of one's own forces and those of the enemy.*

L. Ya. Shepislov developed one pair of formulae for computing the COF in 1981. The first formula applied to forces of the same type:[5]

$$COF = \frac{\sum N_i P_i}{\sum M_j Q_j}$$

For example, N_i could denote the numbers of different types of tanks on one side and P_i a measure of their relative effectiveness. The P_i's were very carefully computed – one type of tank was selected as a standard. Its rate of fire and the size of shells it used were consolidated into a composite measure of its firepower. Similar calculations were made for each type of tank on both sides; by comparing to the standard, measures of relative effectiveness were made for all of the tanks on both sides. Analogous calculations were made for all weapons systems.

The second formula applied to adversarial systems:

$$COF = \frac{\sqrt{\sum \frac{N_i B_i}{\sum K_{ij} M_j}}}{\sqrt{\sum \frac{M_j D_j}{\sum L_{ji} N_i}}}$$

For instance, the N_i could denote different numbers of various types of tanks and the M_j numbers of various anti-tank weapons. B_i and D_j are effectiveness coefficients of each type of weapon (based on firing rate, etc.) The K_{ij} are counter-effectiveness coefficients of each type of tank versus each type of anti-tank weapon (and the reverse for the L_{ji}.) Ultimately, all of the COF values were consolidated into a single measure of the effectiveness of an entire army. However, this final formula is not available in the West at this time, to my knowledge.

The Soviets also developed formulae for assessing the relative strength of their nuclear forces. Major General I.I. Anureyev developed the following formula in 1967[6]:

$$\sqrt[3]{(\frac{Q_1}{Q_2})^2} \frac{\sum \sqrt[3]{u_{i1}^2}}{\sum \sqrt[3]{u_{j2}^2}} \frac{v_{i1}\omega_{i1}}{v_{j2}\omega_{j2}}$$

The Q's denote the total equivalent megatonnage of each side and the u's the fractions of that megatonnage in each weapon. Thus the part of the formula involving the Q's and the u's represent the relative destructive power of the nuclear weapons possessed by each side. The v's denote each side's probability of penetrating the enemies defenses and the ω's denote the proportion of its weapons not destroyed on the ground.

Now let's examine how the Soviets used the COF. Table 2 is an English translation of a page from a Soviet military commander's handbook[7].

CORRELATION OF FORCES AND MEANS

IN THE DIRECTION OF MAIN ATTACK IN THE DEPTH OF IMMEDIATE MISSION						
ENEMY			CORRE-LATION	OWN FORCES		
FORCES & MEANS	TOTAL	PER KM OF FRONT	QUANTI-TATIVE	FORCES & MEANS	TOTAL	PER KM OF FRONT
MECH BNS				MECH BNS		
MEDIUM TANKS				MEDIUM TANKS		
LIGHT TANKS				LIGHT TANKS		
IFC				IFC		
APC				APC		
ARTY & MORTAR				ARTY & MORTAR		
DESTRUCTION OF ARTY PER UOF				DESTRUCTION OF ARTY PER UOF		
ATGMs				ATGMs		
TOTAL DAMAGE TANKS				TOTAL DAMAGE TANKS		
AIR DEFENSE SAMS GUNS				AIR DEFENSE SAMS GUNS		
TOTAL AIR TARGETS DOWNED				TOTAL AIR TARGETS DOWNED		
MOTOR VEH				MOTOR VEH		

Table 2 - Excerpt from Soviet commander's handbook

Soviet commanders were trained to use this table and others like it as battle management tools. They would fill in the size of each side's forces and compute the numbers per kilometer of front. They would then write in the COF for each category in the middle column and use this information in formulating their battle plan and managing their forces as a battle proceeded.

The COF became the principal basis for Soviet "scientific decision-making" regarding the composition and employment of their military forces. It was used as an aid in peacetime strategic planning to determine Soviet force deficiencies and in arms control negotiations, as well as in evaluating options during combat and in predicting the probable outcome of combat and rates of advance of forces.

Conclusions

In spite of all of this careful planning and analysis, the Soviet Union lost the Cold War. What happened? Were the Soviets wrong about the laws of war? Was their mathematics defective? Probably neither. Most analysts today do not regard the Lanchester equations as laws in the precise sense of Newton's laws of gravitation. However, as an approximation they are helpful. For instance, if the COF is decidedly on one side's favor in a battle, that side will probably win, regardless of the stochastic factors. Furthermore, the Soviet's mathematical analyses of the equations were sound. The problem was that the Lanchester equations describe the evolution of a battle, but the US and the USSR did not fight a hot war. Rather, the Soviet Union broke up and its Communist government lost power. That is, the Soviet Union collapsed because of political, social, and economic factors that had nothing to do with the COF – the Soviets had made their decisions in preparation for a battle that never came.

Nevertheless, the story of the COF and its role in Russian history illustrate some important truths about mathematics. First, the influence of mathematics on culture is not always visible, but it can be very powerful. The COF is a concrete example of how a specific mathematical model influenced major decisions in one particular setting. More generally, the COF is a concrete example of how the Enlightenment framework discussed near the beginning of this paper has shaped Western culture for over 300 years. Much of it originated in mathematics. A major priority of the current US administration is rethinking the concepts of deterrence and stability for a post-Cold War world. These concepts are at least quasi-mathematical, so the relationship between mathematics and policy remains important today.

Secondly, the application of mathematics always occurs in a larger context that involves many non-mathematical factors. In this case, this context includes an entire social philosophy (Marxism) as well as the broader world political, economic, and military context. A user of mathematics in such situations will be much more effective if he or she is well acquainted with the non-mathematical factors.

Thirdly, mathematical models may look objective but they may obscure major presuppositions. In this case, the notions that the world is rational, material, and mechanical and that there exist natural laws for human individual and social behavior are implicit in Soviet applications of the COF. Yet the Lanchester equations by themselves give no clue to the existence of this larger framework in which they are being used.

[1] Natural Resources Defense Council, www.nrdc.org/nuclear/nudb/datainx.asp

[2] See, for example, Herman Kahn, *Thermonuclear War* (Princeton. New Jersey, Princeton University Press, 1960) and *Thinking about the Unthinkable* (New York: Morrow/Avon. 1964)

[3] Stillman Drake, *Discoveries and Opinions of Galileo* (New York, Doubleday Anchor Books, 1957), pp.237-8

[4] Alistair C. Crombie, *Styles of Scientific Thinking in the European Tradition: The History of Argument and Explanation Especially in the Mathematical and Biomedical Sciences and Arts*, 3 vols. (London: Duckworth, 1994), p. 1009

[5] James K. Womack, *Soviet Correlation of Forces and Means: Quantifying Modern Operations* (Master's thesis at the U.S. Army Command and General Staff College, 1990) pp.39-40

[6] Judith K. Grange, et. al., *Soviet Measurements of Strategic Balance and Arms Control* (Defense Technical Information Center Technical Report, April 21, 1986), p.18

[7] Ali Jalali, et. al., *Soviet Command and Staff Handbook* (prepared for the US Army BCTP, Springfield, Virginia, June, 1990) p.6-46