

**Making Connections:
Using Analogies to Enrich Understanding of Mathematical Ideas and Biblical Truths**

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Let every good and true Christian understand that wherever truth may be found, it belongs to his Master.
- Augustine

According to *Principles and Standards for School Mathematics* (NCTM, 2000), and the recommendations of most other mathematics professional organizations, one of the overarching standards for mathematics learning at all grade levels is “connections.” Through an instructional emphasis on the interrelatedness of mathematical ideas, students not only learn mathematics but also “learn about the utility of mathematics.” Students should see the subject not as a collection of separate strands, but rather as an integrated field of study.

Types of Mathematical Connections

This approach encompasses three components. One facet of *connections* enables students to recognize and use connections among mathematical ideas. Many problems and classroom activities can be productive for connecting ideas from measurement, data analysis, geometry, algebra, calculus, and/or number theory. Teachers can encourage this thinking by frequently asking questions such as “How is this problem or mathematical topic like things we have studied before?” This is always a good question for teachers to ask and helps students relate and attach new information to previous knowledge.

The second facet enables students to understand how mathematical ideas interconnect and build on one another to produce a coherent whole. As they develop this view of mathematics as a connected and integrated whole, students will be less likely to view mathematical skills and concepts separately or perceive mathematics as an arbitrary set of rules. For the teacher, this means it is crucial to help students link procedural knowledge with conceptual understandings. When we ask, “Why?” or “How do you know?” we give students opportunities to attach reasons to their procedures and to use mathematical ideas to justify and support other mathematical ideas.

The third facet of *connections* enables students to recognize and apply mathematics in contexts outside of mathematics. These connections can be to other subject areas and disciplines as well as to students’ daily lives. It is important that students experience mathematics in some context and, of course, mathematics permeates areas such as the natural and social sciences, business, and even the arts and humanities. For example, students should be able to use their knowledge of data analysis and mathematical modeling to understand societal issues and workplace problems in reasonable depth. Thus, we hear much today about the need for “real-life” applications in the process of teaching mathematical concepts and skills.

It seems clear from research that when students connect mathematical ideas, whether to ideas within or beyond the realm of mathematics, their understanding is deeper and more lasting. In fact, understanding necessarily involves *making connections*. Ideas are never understood in isolation; previous knowledge is necessary for making sense of any new idea.

Therefore, it seems reasonable to conclude that when we utilize this third type of connecting by encouraging students to compare, contrast, analyze, or apply ideas from two different domains, we have increased the likelihood that they will develop deeper understandings within both domains. Some of us are old enough to remember Bloom's taxonomy of intellectual thought. In this hierarchical theory, thought processes that involve application, analysis, synthesis, and evaluation are considered higher levels of thinking and learning than simply knowledge or comprehension. I believe that by asking ourselves (or our students), such questions as "How is Idea/Term A from Domain X similar to or different from Idea/Term B from Domain Y?" or "How can we apply this idea (from one domain) to that idea (from a different domain)," a richer and more complete understanding of *both* domains can occur. One area sheds revealing light upon the other and new insights may be gained that were previously undiscovered. And, such processes need never cease since there are no final or complete answers to such "connecting" questions. The insights I gain from such a study or analysis may be very different from those of a colleague. Therefore, I believe this desired outcome can result when we attempt to integrate mathematical ideas and terms with biblical ones.

Math-Bible Integration

This idea of integrating content knowledge with biblical truths or theological beliefs is one that Christian teachers have been grappling with for many years and will, no doubt, continue to do so. It is a foundational reason for the very existence of the organization ACMS and its biennial conferences and various projects. Thus, "connecting" or "integrating" faith and learning is the mandate of all Christian educators.

There seems to be many approaches to such integration and teachers often ask, "What is the right or best way to do it?" Is it by using the Bible as a math textbook? Is it by assigning spiritual significance to biblical numbers? Is it by putting word problems in a Christian context? Probably not. How about starting math class with a devotional thought, a verse of scripture, or a prayer? These latter practices I have done and continue to do, and I believe they do have value within the Christian learning environment. Yet, I am left unsatisfied with these practices alone because they are not necessarily "integrative" in nature. In such cases, it can easily seem as though one thing is artificially "tacked on" the other. No true integration is thereby achieved. My personal view is that the best way to integrate Christianity with our discipline involves taking a holistic perspective where God is seen as the foundation of all knowledge and therefore, anything from His creation reflects back to Him (see Augustine quote at beginning of article).

Many thought-provoking presentations and papers on this idea have been given in past conferences, but time and space do not allow me to elaborate on that view here.

Using Analogies and Personal Perspectives

I do, however, want to discuss one specific activity that we all have likely used in our own attempts to link our faith with our discipline – the use of analogies. The term *analogy* has its root in the idea of *proportion* or *relation*. I am taking a broad view of the term in this paper and using it to mean a similarity, a correspondence, or parallel. It is the comparison between things that may be alike in some way but may be otherwise unlike. It does not imply an exact equivalence. From logic, we know that analogy proceeds on the basis of similarity of two situations. If the similarity is close enough, one may conjecture that a conclusion that holds in one situation should hold in the other. However, we must also be cautious to not extend analogies too far. This use of analogies is an ancient form of teaching. Samuel Johnson wrote, “To illustrate one thing by its resemblance to another has been always the most popular and efficacious art of instruction.” Even the profound stories of Christ we call “parables” are a form of analogy.

I still recall some of my earliest and most primitive thoughts in this area. While in junior high school during the “modern math” era, I was taught definitions or descriptions (I learned much later what “undefined terms” meant) of the basic abstract building blocks of geometry: point, line, plane, ray, half-line, segment, etc. As I was taught that a line has only one dimension (length), extends infinitely in two directions, and has no endpoints; that a ray has one endpoint and extends infinitely in only one direction; and a segment has two endpoints, my thoughts returned to my earliest Sunday School training. Thinking in terms of time, I likened a line to God who was in the beginning (pre-existent), who lives in the present, and who has no end. He is from everlasting to everlasting, “...the Alpha and the Omega,...who is, and who was, and who is to come...” (Rev. 1:8 NIV). I thought of the ray as humankind (or perhaps the angels). As individual beings with a soul, we have some beginning point of creation but our soul will never cease to exist. Our created life had a beginning (endpoint?), but our time extends infinitely from that point on through eternity. A segment seemed to be like any other (nonhuman) creation of God – the earth, for example. It has a beginning and has or will have an end.

My Geometry Analogies (time/existence)

line ↔ God

ray → people

segment — other creation

Although these comparisons were quite simplistic and really didn't make the baffling concept of "the infinite" any more comprehensible to a junior-high school student, my self-made analogies did allow me to feel more "comfortable" with the new material. I even wondered if children, like me, who were raised in Judeo-Christian traditions might have some advantages over other children in understanding such abstract ideas of mathematics as these. I wouldn't now argue that that is the case, but as for me, because I had encountered somewhat similar ideas previously and, even though I still could not fully fathom the magnitude and implications of such big ideas, I could accept them for the moment, store them away in my mind, and from time to time, draw them out to re-examine and to build upon.

I've since discovered that we all bring our history to a new situation and whatever training and patterns of thinking that are in our background, we use to make sense of and to help us describe novel circumstances. Those of us trained in logic or mathematics naturally tend to apply our paradigms of thinking in other contexts as well. For example, a physicist friend of mine was speaking about Jesus' statement that "true worshipers will worship the Father in spirit and truth" (John 4:23 NIV). As to how we might know we are worshiping "in truth," he said, "It's not the absolute value; it's the sign of the derivative (the slope of the function) that matters. Are we going in the right direction?" Probably, only part of his audience had a good understanding of what he was talking about, but all of us have likely done something similar to this because some analogy or metaphor makes perfect sense to us.

I am convinced that the task of "analogy-making" can be valuable not only for me but for students of mathematics at all levels. Traditionally, in college mathematics, we seem to reserve the challenge of "making math-Bible analogies" for those students who are in upper level mathematics courses. There is a certain advantage to engaging those students in thinking "out of the box" because they have already attained mathematical understandings and appreciations that non-math students have not. However, such activities need not be limited to only the strongest mathematics students. I have taught college courses such as Finite Math, Introductory Calculus, Algebra/Trigonometry, and Mathematics for Elementary Teachers. Students in these course are not math majors and, in fact, some of them have a strong "math anxiety." Nevertheless, all of them, I believe, can benefit from engaging in some kind of thinking activities that foster connections between mathematical ideas and biblical ones.

A Thinking/Writing Assignment Example

Even though the students often do not have a strong math background or interest, I sometimes give them a writing assignment based on their personal search for a math-Bible connection. I usually provide an example or two and some guidelines. Then, I grade them, not on whether their analogy is right or wrong, but on how well they followed the guidelines of accurately discussing the mathematics and linking it in a reasonable way to scripture. Students often start with a word that appears in scripture and also has a mathematical meaning or usage as well. This invites them to use a math dictionary and Bible

concordance. Of course the limitation of focusing on specific scriptural words (rather than broader ideas) is that you are constrained by the translation or version you are using.

Let me present a simple example that illustrates the kind of thinking process I am describing. As you will see, I offer more questions than answers. However, I make no apologies for this because I believe good education generates more questions than it does answers; otherwise the whole process comes to an end! I might introduce the thinking/writing activity to the students in the following way. The first two verses of Romans 12 read:

Therefore, I urge you, brothers, in view of God's mercy, to offer your bodies as living sacrifices, holy and pleasing to God – this is your spiritual act of worship. Do not conform any longer to the pattern of this world, but be transformed by the renewing of your mind. Then you will be able to test and approve what God's will is – his good, pleasing and perfect will. (NIV)

Many words used in mathematics also have meaning in everyday English and even appear in scripture. Sometimes these usages derive from similar meanings and sometimes they are quite different. One of these words appears in Romans 12:2, “transformed.” In mathematics, often but not exclusively in the area of geometry, we study types of transformations. Some of these transformations or mappings are congruence motions (isometries) such as translations, reflections, rotations, and glide reflections. Others do not preserve distance and shape, such as dilations.

Part of the power and value of mathematics is its ability to aid us in understanding the created world in which we live. Because analogies can sometimes help us better understand complex ideas, it is interesting to think about comparisons between the mathematical term “transformation” and the concept of a spiritual “transformation” or change that Paul is talking about in this passage. This must be done, of course, with the realization that analogies often can take us only so far in comparing two things and that there are limits to any similarities we may observe.

The following questions might challenge us to examine and to better understand the notion of “being transformed” in the biblical sense:

1. How does this transformation Paul mentions compare to the geometric transformations defined in mathematics?
2. What happens when we are “transformed by the renewing of our mind?”
3. When we are spiritually transformed, what remains invariant and what is altered?
4. Are there aspects of spiritual transformations that remind us specifically of translations, reflections, rotations or other geometric mappings?

Here are some possible ideas for comparisons we could make:

Rotation analogies: A spiritual transformation changes our direction just as a rotation can alter the direction in which a geometric object is pointing. Think about an arrow, like a hand on a clock face, pointing at 6 o'clock. With a 30-, 90-, or 120-degree rotation, the arrow/hand is pointing in a different

direction. In a spiritual transformation, the analogy might be that the power of Christ rotates us 180 degrees so that we are headed in exactly the opposite direction than before we were changed. The conversion of Saul (to become Paul) is perhaps the most vivid biblical illustration of this total change in direction. The most basic description of spiritual conversion is that of “turning” to God. Tryon Edwards has written, “Conversion is but the first step in the divine life. As long as we live we should more and more be turning from all that is evil, and to all that is good.” My colleague William C. Ringenberg in his new book, *Letters to Young Scholars*, says, “If turning is the central idea in conversion, the central element in turning is the transformation less of the intellect or even the belief system than of the will. A Christian convert is a person who wills to will God’s will.” Therefore, obedience becomes the new imperative of a life that has “rotated” to God’s direction.

Translation or Reflection analogies: What could we say about becoming more and more a reflection or image of Jesus Christ through the process of being spiritually reborn/transformed? Remember a reflection transformation can change the orientation of an object, that is, its relationship to objects around it and the relationship of parts of the whole to each other. (This is an intuitive description for a term not so easy to define.) Does this happen in a spiritual transformation? How does this relate to 2 Corinthians 3:18 which says, “And we, who with unveiled faces all reflect the Lord’s glory, are being transformed into his likeness with ever-increasing glory, which comes from the Lord, who is the Spirit.”?

Another important idea from transformational geometry is that of “invariant.” When we are spiritually transformed, what about us does **not** change? Our bodies? Remember, they are to be a “living sacrifice” (Romans 12:1). Our basic personalities? Paul says in Ephesians 4 that we are to put off the old self...and put on the new self. So, when our spirits are transformed, is there any similarity between the old and the new that it has been “mapped into” or replaced by? Or is this even a valid analogy?

Perhaps you can think of your own questions and possible answers as you think of your own personal spiritual experiences. Since being spiritually transformed, how are you different? How are you the same?

After an introductory example and thinking exercise (such as the discussion above), I might pose the following assignment to students:

Choose a term or concept that has both mathematical and biblical meaning and discuss any parallels you see. In other words, compare and contrast the two meanings/usages of the term. Your reflection paper may take the form of an analogy (how is some term/idea in math similar to some biblical truth?) or the form of a brief devotional. You may need to consult a math dictionary and Bible concordance.

Perhaps you might disagree with some of the specific spiritual assertions made in the earlier analogy. That is fine. You can create your own set of questions and answers. Ultimately, the main purpose of the exercise is not so much the outcome as it is the thinking process itself. The real goal is to

take ideas that may seem very familiar to us and examine and describe them in unfamiliar ways through the vehicle of making connections, metaphors, or analogies. This process reveals something about the depth of understanding of both the mathematical and spiritual ideas.

In some cases, students' reflections are not genuine analogies but instead, a mathematical term or idea is used more or less as a "point of entry" into contemplating a spiritual truth or for the genesis of a devotional thought. Sometimes a biblical passage is "mathematized" (by this I mean "expressed or explained in mathematical language"). For example, in explaining Matthew 6:19-21, one student thought of it this way: treasures in heaven = (service on earth) x (constant). For this person, the linear equation was a way of conceptualizing the idea that the more effort we put into serving Christ on earth, the more treasure we will have stored in heaven. I sometimes encourage students to think about spiritual ideas using mathematical language or to take mathematical language and attempt to apply it to some biblical truth. Following is an example of using the idea of reflection as a "mirror" image as a "point-of-entry" for devotional introspection.

Seeing God in the Rear-view Mirror

"...Not one of all the good promises the Lord your God gave you has failed. Every promise has been fulfilled; not one has failed." Joshua 23:14 NIV (also Joshua 21:45)

We do not have enough opportunities for reflection in our lives – times to look back and see God in retrospect. God's grace is often seen more in reflection than it is in the heat of the battle. It is in looking back that we realize His faithfulness and the fact that His grace was sufficient. It is in looking back that we calculate the effectiveness of His power in our moments of weakness. It is in looking back that we find the assurance that His Spirit was our guide during those times we had difficulty seeing the signposts along the way. We might think of this as seeing God in the rear-view mirror. We see out of the windshield of our lives more clearly when we are able to look in the rear-view mirror and see God in our past. The future is less frightening when we reflect on the fact that not one of all of the good promises the Lord has given us has failed. If we look at the right-hand outside mirror of our car we will see the words "Objects in the mirror are closer than they appear." This represents an example of a size change transformation or dilation. The image is smaller than the pre-image; the object appears to have shrunk. Well, it works that way with God too. It is in seeing God's presence in our past that we know He is nearer to this present moment than the circumstance might seem to suggest. We need to find the time to quiet ourselves and reflect on God's goodness in our past and in our present. His faithfulness is guaranteed. His grace is never ending. He is closer than it may appear. – (adapted from Kenneth Heer)

Below is an excerpt from Nathan's (an elementary education major) reflection on Ephesians 3:17-21. He grapples with the puzzling issues of theology in a playful manner.

"Get out your rulers, class; it's time to measure God's love!" Thankfully, none of us have ever heard our teacher start class in this way, because, surely, it would be quite a daunting assignment! Paul describes Christ's love using four different dimensions: wide, long, high and deep. This seems a little repetitive, but perhaps it is because Christ's love is simply that huge! You would need something a little larger than a yardstick, even larger than a tape measure, to measure God's

love! It is almost as if Paul can't make up his mind. He seems to describe Christ's love in a *measurable* way, but then states that it *surpasses* knowledge. Then, he prays for believers to be filled to the *measure*. And then again, states that God can do *immeasurably* more than all we can imagine! So which is it?

Despite this inconceivable quality, Paul puts forth a passionate prayer for a worthy goal. Our human minds can never comprehend God's love, but Paul fervently prays for a small glimpse of that love. He wants us to have the taste-test that leaves us yearning for a bigger bite. He wants us to be filled with the fullness of God as much as possible – a fullness that is not measurable. May we desire to know God and His love as much as possible in our natural, human, limited state. We cannot measure it with our human ways of measurement, but let's pray with Paul for help to grasp at least an inch or two.

Here are some additional "idea-generators" that I have provided students or ideas that they have found to write about. You can vary the possibilities depending on the mathematical sophistication of the students. Most or all of these are certainly accessible even to "non-math" inclined students.

Ideas for Connecting Mathematical Terms or Ideas to Biblical Ones

Measuring volume; dimensions: A favorite passage for students to write about is Ephesians 3:17-19 to discuss measurement or dimensions. One student said that Christ's love is total and reaches every corner of all our experiences. It is *wide* - it covers the breadth of our own experience, and it reaches out to the entire world. It is *high* – it rises to the heights of our celebration. His love is *deep* – it reaches down to all our disappointments, despair and sadness. Students are often baffled and challenged by what seems to be the use of four dimensions in this passage. A colleague of mine once commented that there is one location where all four dimensions make sense. If you imagine yourself embedded in the interior of an object such as a prism, you might be interested in the distance to the edge in front of you, to your side, above you, and below you. Is this an implication that we are completely surrounded by Christ's love (that it is all-encompassing)?

Pattern – Romans 12:2, Romans 5:14 (Can patterns be negative/undesirable things?)

Limit – Psalm 147:5 ("...his understanding has no limit"); Psalm 119:96; I Thessalonians 2:16; or "how close can we get to God?" In *The Purpose Driven Life*, Rick Warren states, "You are as close to God as you want to be."

Equality – Isaiah 46:5-7

Eternity, eternal, infinite – Ecclesiastes 3:11,14; Psalm 100:5; Isaiah 9:7 (a good opportunity to research the history of the idea of infinity)

Measurement – Mark 4:24

Reflection – 2 Corinthians 3:18

Fractions, unity, division – I Corinthians 12:12-26

Addition – Matthew 6:33

Multiply – Genesis 1:22; 2 Peter 1:2

Prayer of two or three (total is greater than sum of its parts)

Higher dimensions – see Neuhouser, David, "C. S. Lewis and Mathematics" at www.tayloru.edu/cslewis and click on the articles and reviews; or see "Higher Dimensions and Christianity" in *Open to Reason* (2001).

Foundations (math and spiritual) – Isaiah 28:16

Change (calculus); constant; variable – e.g. Isaiah 26:3

Certainty (probability) – Joshua 23:14

Function - Romans 5:20 (where sin increased, grace increased more)

Abundant, deficient, perfect (number theory)

*Infinite limits – Philippians 3:10-14

*Paradoxes – Colossians 1:16-19; I Corinthians 15:27

*Sequences – James 1; Romans 10:13-15; the genealogies

*Logic – James 5:12; Matthew 12:30; I John 1:9; Jeremiah 33:3

*The nature of proofs – John 20:25; I Peter 1:7-8; I Corinthians 2:5; Mathew 27:42; Luke 16:31

* see Brabenec, Robert, “Using Mathematical Concepts to Illustrate Scriptural and Spiritual Ideas.” *Proceedings of the Fourth ACMS Conference* (held at Wheaton College, May 25-28, 1983)

Benefits of the Assignment

A writing assignment such as the one described above provides several benefits. First, it helps me, the professor, get better acquainted with each student in my class. I learn something about each student’s faith, beliefs, and/or spiritual experiences.

Secondly, it provides an informal assessment tool as I ascertain a bit of the student’s level of understanding of some mathematical concept. Sometimes I see a depth of knowledge that was previously unnoticed, and sometimes I detect misconceptions or incomplete knowledge. In one case, a student wrote that a math problem has only one answer. This was particularly discouraging to me since we had just spent several weeks earlier in the course engaged in problem solving and noting that many problems had multiple solutions. Another student wrote that to multiply always “makes bigger.” Was this a temporary lapse in memory, a genuine ignorance of integers and rational numbers, or perhaps a conscious omission to conform the math term to the biblical meanings of Genesis 1:22 or 2 Peter 1:2? One student, mentioned previously, expressed the truth of Matthew 6:19-21 as: treasures in heaven = (service on earth) x (a constant). Should we be more precise about what kind of constant, since not all values would increase the product?

I was also puzzled by the student who wrote that “mathematics is ruled by an order of operations.” The student then described in wonderfully inspirational ways how the four basic arithmetic operations relate to the process of living a Christian life (in a nutshell: “Christ was added to my life, He took away my sins, He multiplied Himself in me, and now I divide and share that love with the world”). The problem, however, was that the student insisted that the correct order of operations in a mathematical calculation is: addition, subtraction, multiplication, and then division. Such errors or misconceptions provide me with opportunities to give instructive feedback to individual students or sometime to initiate a class discussion on the relevant topic.

Thirdly, I see the thinking/writing assignment as valuable because sometimes students discover new insights about a mathematical idea they have had to investigate or about a biblical truth. Is this activity the answer for teaching mathematics from a Christian perspective? Is it true integration of faith and knowledge? Probably not. At best, it is incomplete. It might better be classified as an activity that provides an “intersection” between biblical truth and the domain of mathematics. Even though students’

work often lacks the depth of understanding (both mathematically and spiritually) that instructors might desire, I believe this type of activity can foster a disposition in students to continue their personal search for observing many types of connections in their daily lives. The ultimate success of any such exercise is achieved if and when students can develop a stronger connection between their knowledge and their Christian faith and apply such enriched understandings to all of life's experiences.

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