

## A REACTION TO THE POYTHRESS PAPER

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My assignment is a rather unusual one for me since it asks that I react to a metaphor, for, indeed, that is what the Poythress papers are. Rather than writing in heavy, prosaic, philosophical style, Vern has seen fit to write metaphorically about the nature of mathematics. The change is unusual, attention-gaining, and most welcome.

A natural question to ask is "How does one recognize a good metaphor?" Suppose I say: "The universe is God's assembled tinker-toy kit and mathematics is the arrangement and the number of holes on the circular pieces." Is this a good metaphor? Certainly not. But what is a good metaphor?

Here we are getting into very deep water to say the least. What I say will be superficial and "off the cuff." Some of today's most talented philosophers are working on the hermeneutical problem of analyzing metaphor and constructing a general interpretation theory.

The structure of a metaphor is something like this. We have some difficulty in defining or characterizing a phenomenon or entity A and we use a secondary entity B which is closer to our everyday experience in some sense and we say (directly or indirectly) "A is B." We say

John is a lion or

The Universe is God's choral poem or

Mathematics is its rhyme.

In doing this we present the reader with a puzzle which challenges him to see in which ways A is B. Or we help create a mental picture in one's mind which uses B to better characterize A. To be a good metaphor it must have some of the following properties:

1. it is enlightening
2. it is delightful
3. it is surprising
4. it invites some controlled analysis of what the connection between A and B may be.
5. it has a personal, direct quality which compels one's attention.
6. it elicits response and enjoyment.

Notice that we are not saying that A is identical with B. That would be dull and boring (as a metaphor). We are saying that A is like B in some sense. Max Black in his interesting book Models and Metaphors says that we use B to put a filter on A so as to highlight and organize some facet of the complex A for our scrutiny and enjoyment.

But that is not the end of the matter for we go on in our analysis. We use the metaphor to draw conclusions somewhat as follows:

A is B. But B has property P. So A (may) also have property P.

Inversely we proceed as follows. There is some disagreement about A and its properties. Some say A has property P. Now A is B. But B does not have property P. So A may not also.

Obviously such reasoning and analysis using metaphor is hazardous and treacherous. We mathematicians are trained to avoid such situations at least when we display our final products! So much the worse for ourselves.

To return to my assessment of the Poythress metaphor about mathematics; namely that it is the rhyme in God's choral poem. I find that it is a good one since it meets many of the descriptions that I listed. It is delightful, surprising, and does invite controlled analysis.

Let us see, however, if the metaphor is used properly by looking at some of the "consequences" that are drawn about the nature of mathematics. As I see it Poythress uses his metaphor to describe mathematics in the following ways.

1. Mathematics is a distinct science and is not reducible to any other sciences. Thus mathematics is not reducible to logic (logicism), it is not a language game (formalism), and it is not pure mental construction (intuitionism).
2. Mathematics contains truthful, a' priori propositions.
3. Mathematics has content and deals with the properties of identity and difference. It deals with the properties of qualities, with properties of space and number.
4. Mathematics as a discipline has fuzzy boundaries and as such it is difficult to characterize mathematical activity.

I liked these findings on the nature of mathematics and I agree that they are faithful descriptions of the discipline of mathematics. Of course, there are limitations that a metaphor has in giving a complete description of the principal component. One would also like to stress other properties of mathematics. These might include the fact that mathematics has a long and illustrious history, that it arises from cultural efforts of man, that it has astounding power in dealing with the spacial and numerical aspects of the creation order, and finally that it has rare beauty and elegance. These aspects might arise as the result of other metaphors to be constructed later.

One thing that I did see more clearly by reading the Poythress paper is that the positions of logicism, formalism, and intuitionism might be better viewed not as reductionistic in nature but better as metaphors on the nature of mathematics. They each give an enlightening view of a facet of mathematical activity and could better be viewed as metaphorical rather than definitive descriptions.

Let me make one more comment on the reasoning in the Poythress paper. On several occasions Poythress uses what I will call the Benecerraf Move. Paul Benecerraf uses it to show that numbers are not sets. It goes like this:

To refute the assertion that  $A$  is a subset of  $B$  proceed as follows:

Suppose  $A \subseteq B$ . Then  $A = C$  where  $C$  is some subset of  $B$ . Now person  $A$  considers  $C$  as  $C_1$  while person  $B$  considers  $C$  as  $C_2$ . But clearly  $C_1 \neq C_2$ . So  $A \neq C_1$  or  $A \neq C_2$ . But we have no reason to prefer  $A$  or  $B$ 's formulation. So  $A \subseteq B$ .

The hole in the argument is obvious.  $A \neq C_1$  or  $A \neq C_2$  but one of these may be true or there may be a  $C_3$  so that  $A = C_3$ . The force of the argument arises from our agreeing that we have no preference for  $A$  or  $B$ 's formulation and we can't think of other alternatives. Thus I find the Benecerraf Move somewhat unconvincing. Poythress uses the Move to show that mathematics is not reducible to logic or psychology, etc. I agree with the result but not with the argument. As a final point Poythress uses the Move to say that nothing is identical to anything else - which is certainly a harmless statement!

To summarize, I liked the Poythress metaphor. I believe that Vern has shown us a new way to tackle the complex task of describing the mathematics we love. We are in his debt for it.