Using Java and HTML for Linear Algebra Instruction

Jonathan R. Senning
Gordon College

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Abstract

This paper addresses some of the issues involved with using the HTML, JavaScript and Java to develop and serve a sequence of laboratory modules for use in teaching linear algebra. Attention is paid to the rationale for this approach as opposed to the more traditional approach of laboratory exercises executed using MATLAB or some similar computational tool. Several methods to display mathematics with HTMLare described. Some implementation detail and a brief description of the HTML and Java based *Linear Algebra Visualization Assistant* (LAVA) is presented.

1 A brief history of reform in linear algebra education

In 1987 the *International Matrix Group* was formed for the purpose of supporting endeavors which used linear algebra. In 1989 this group was replaced by the *International Linear Algebra Society*[1]. The ILAS seeks to support existing linear algebra groups throughout the world, and to identify and fill any gaps that exist between these groups. The Education Committee of the ILAS is concerned with supporting innovated efforts to each linear algebra and along with the National Science Foundation (NSF) it supports the ATLAST project (described below).

The Linear Algebra Curriculum Study Group was formed in January of 1990 and enlarged in August of that year. The LACSG generated a list of recommendations for first courses in linear algebra [1]. The recommendations are:

1. The syllabus and presentation of the first course in linear algebra must respond to the needs of the client disciplines.

[1]*http://math.technion.ac.il/iic/index1.html*

2. Mathematics departments should seriously consider making their first course in linear algebra a matrix oriented course.
3. Faculty should consider the needs and interests of students as learners.
4. Faculty should be encouraged to utilize technology in the first linear algebra course.
5. At least one "second course" in matrix theory/linear algebra should be a high priority for every mathematics curriculum.

A project to *Augment the Teaching of Linear Algebra through the use of Software Tools* (ATLAST) was originally funded by the ILAS and received further funding from the NSF. It was basically an effort to provide a collection of software tools and teaching materials to allow instructors to implement the recommendations of the LACSG. Most of the work done by the ATLAST project was carried out at sixteen NSF-supported workshops during the summers of 1992–1996. Early workshops dealt with the construction of *m-files* to use with MATLAB while later workshops focused on creating lesson plans that used MATLAB and the previously created m-files. Last year a book was published that contains edited material that was developed during the ATLAST workshops [2].

2 MATLAB, Octave and symbolic algebra packages

This section briefly describes several software packages that can be used for linear algebra instruction. While only a few of the more well known packages are mentioned here, there are many more packages available from textbook publishers and from individual authors.

Probably the most widely used computer program for linear algebra is MATLAB (The MathWorks, Inc.) MATLAB was originally a user-friendly front-end to libraries of FORTRAN subroutines (LINPACK and EISPACK). It has grown into a self-contained programming language and supports sophisticated two- and three-dimensional graphics. It is a numerical program as opposed to a symbolic program in that it works with floating point numbers and is not capable of symbolic operations.

A partial clone of MATLAB, Octave was developed by John Eaton at the University of Wisconsin, Department of Chemical Engineering. It is a free software package for Unix-based computers (and soon will be supported on Windows 95 and Windows NT machines). It has a great deal of the functionality of MATLAB and is able to run many m-files that were designed for MATLAB. However, it does not support the interactive graphics and animation available with recent versions of MATLAB.

Included in this category of symbolic algebra packages are programs such as Mathematica and Maple. Like MATLAB, these programs have sophisticated graphical capability and can do advanced numerical operations on matrices. However, unlike MATLAB, these programs can do symbolic manipulations on matrices.

3 Potential for web-based tutorials

Clearly, programs such as MATLAB, Mathematica and Maple provide the most complete, robust and powerful high-level software tools for mathematics in general and linear algebra in particular. They are popular and therefore an abundance of literature exists to help educators and students use these programs. Many textbooks include information on how to use such programs and many schools have computer laboratories equipped with these programs.

However, these programs have much more functionality and the associated complication than is required for many concepts introducted in a first course in linear algebra. In addition, there are all the issues associated with licensing, networking and platform compatibility to contend with.

A well-designed web-based tutorial can avoid some of these difficulties. Keep in mind that the purpose of this tutorial is not to provide the functionality of a program such as MATLAB, but instead to be used by students to help them understand basic concepts. By making the tutorial available on the web students can use it when it is convenient for them; in a lab during the day or evening and also from their home or dorm room if they have access to the school's network. It is also possible to integrate the tutorial with e-mail and/or a newsgroup so that the students can interact with the instructor and other students.

4 Displaying mathematical notation on the web

One of the strongest impediments to using HTML and the web for mathematics instruction and information dissemination is that current incarnations of HTML do not support mathematical typesetting. It was originally planed to be included in HTML 3.0 but has been postponed at least until HTML 4.0. There are several reasons for the delay. One of the more interesting reasons is that there is a strong, conscious effort to ensure that all HTML can be interpreted by special browsers for people with disabilities, such as blindness.

Until a version of HTML is released that supports mathematical typesetting, and until browsers exist that can display this version of HTML, an individual wishing to include mathematical expressions in an HTML document does have some (albeit limited) options:

Use the standard character set: This is how typewriters without special symbols and older text-based terminals are used to display mathematical expressions. While this works, it is very ugly and confusing. Also, since it depends on proper text spacing, the `<PRE>` and `</PRE>` tags must bracket the expressions.

Create images of expressions: Another option is to convert expressions from word-processor documents to GIF or other types of images and display them as images in the HTML document. This can be done several ways, either by converting expressions one-by-one or using programs such as LaTeX2HTML to convert entire documents (in this case, documents created using LaTeX). One major problem with this method is that the font size and background color of the expressions are fixed, while the browser may be configured to use a different font size or background color.

Use Browser Plugins: Plugins, such as nDVI[2] by K. Peeters or IBM's techexplorer[3], can be used with Netscape or Internet Explorer to display entire TeX and LaTeX documents or, in the case of techexplorer, individual expressions formatted using TeX or LaTeX.

Use Java Applets: Several different Java applets are freely available that can be used to display mathematical expressions. Two of these that are available from the Geometry Center[4] are WebEQ and IDVI. **WebEQ**[5] processes expressions formatted with WebTeX, a language based on the math commands from LaTeX and in the near future is expected to also process expressions formatted with MathML, the math markup language that is proposed for inclusion into future versions of HTML. **IDVI**[6] is a Java applet that displays entire DVI files. These are the device-independent files produced by TeX and LaTeX processors. Thus, IDVI will display exactly what would be printed out by a laser printer. One disadvantage of IDVI is that your server must be prepared to serve any fonts that the DVI file may use, adding significantly to the server diskspace requirements.

A more complete list of options can be found at the World Wide Web Consortium's math web page[7]. W3C also has a page devoted to integrating mathematical markup into web documents[8] and has released a working draft of a mathematical markup language, MathML[9].

[2] *http://norma.nikhef.nl/~t16/ndvi_doc.html*
[3] *http://www.ics.raleigh.ibm.com/ics/techexp.htm*
[4] *http://www.geom.umn.edu*
[5] *http://www.geom.umn.edu/software/WebEQ*
[6] *http://www.geom.umn.edu/java/IDVI*
[7] *http://www.w3.org/Math*
[8] *http://www.w3.org/pub/WWW/MarkUp/Math*
[9] *http://www.w3.org/TR/WD-math*

5 LAVA: Linear Algebra Visualization Assistant

The two main goals for LAVA are to

1. provide an easy-to-use tutorial for some concepts covered in an introductory linear algebra course with the intention that coverage of further topics will be included in the future, and to
2. provide platform independence and network accessibility.

Work on LAVA began during the summer of 1996. At that time I decided to use Java to create interactive applets that would provide much of the geometrical understanding of the concepts to be presented. In fact, the desire to provide a visual, geometrical understanding of the underlying concepts drove the selection of topics that LAVA includes. In all cases, they are topics that are easy to understand in two or three dimensions.

Topics planned for the first version of LAVA are

1. Elementary row operations and row-reduction of a matrix,
2. Linear independence,
3. Linear transformations,
4. Eigenvalues and Eigenvectors.

Other topics can be easily integrated into LAVA; possible topics include an introduction into bases (including change of basis) and to orthogonalization.

LAVA will be available on the web around the start of 1998. However, snapshots of the package during development can be found at *http://www.cs.gordonc.edu/~senning/lava/*. Once LAVA is complete, instructions on how to download the entire package can be found at this URL.

The material format in LAVA is common for on-line tutorials. New material is presented and then the student is asked one or more questions based on this materal. These questions typically fall into one of two catagories: *content questions* that cover the material just presented, and *review questions* that cover material the student should already know. The questions are multiple choice, and every possible answer (both correct and incorrect) supplies additional information.

Figure 1 shows a typical LAVA page. The area on the left is used for navigation between major topics and sections within a topic. The large area on the right is the content section

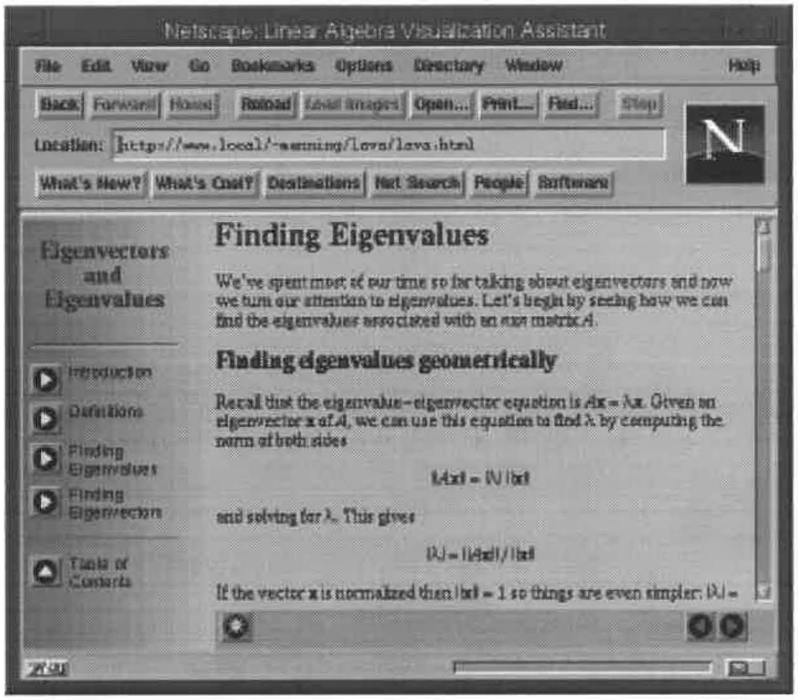

Figure 1: Typical LAVA Page

where the topic information is displayed. At the bottom on the right is a button bar that allows the user to navigate sequentially within a section and also has a button to report any problems to a local site administrator.

Figure 2 shows the pop-up Netscape window with the interactive tool associated with the section on elementary row operations. This tool can work with systems of two or more equations, but only systems of two equations have the associated graphical display. In this case the graph of the equations is shown and the graph changes as the row operations are performed. The object of this tool is to allow the student to see that row operations do not change the solution of the system, but ultimately replace the system with an equivalent one that can be solved by inspection.

Figure 3 shows the pop-up Netscape window with the interactive tool for exploring eigenvectors and eigenvalues. The shorter line (in blue on the screen) is the normalized vector x; the circle is the unit circle. The direction of x is controlled by the mouse when the (right) mouse button pressed. The longer line (in red on the screen) is the vector Ax; the ellipse is the image of the unit circle under the transformation specified by matrix A. Students are led through several exercises where they use this tool to explore the eigenvalues and eigenvectors of a matrix.

Mathematical equations are displayed using small, inline GIF images and HTML tables. Matrices are well suited to display using tables. The only difficulty is putting the paren-

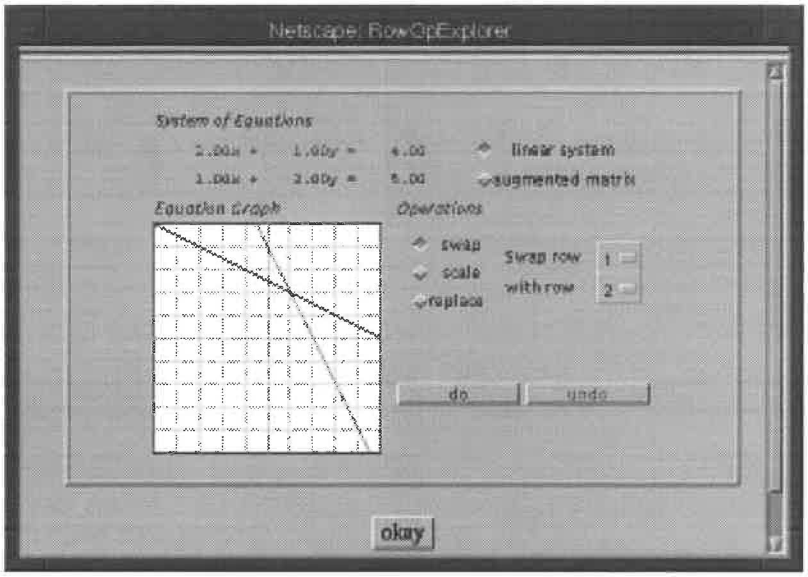

Figure 2: RowOpExplorer window

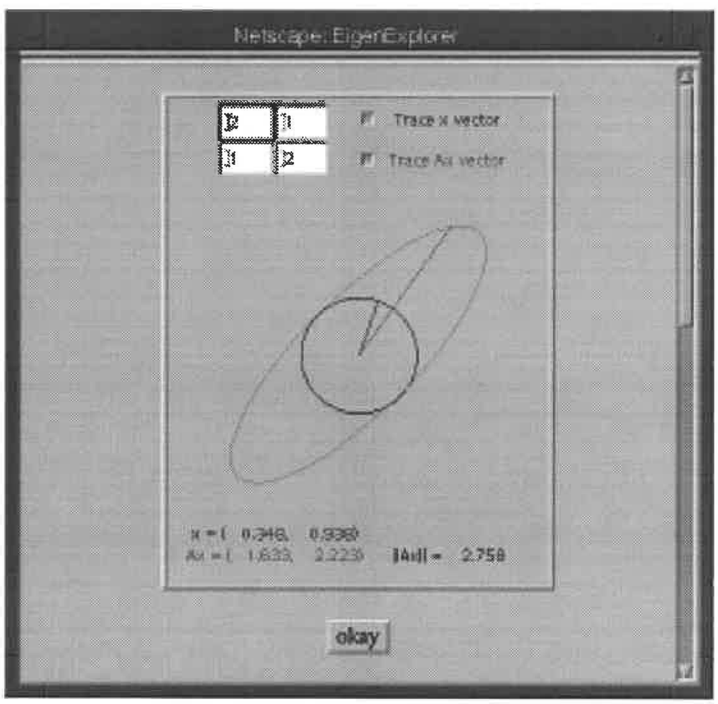

Figure 3: EigenExplorer window

thesis around the array of numbers. HTML tables can be tedious to work with, so LAVA uses a *preprocessor* to expand statements like `showMatrix(1,2,3,4)` to an HTML table (with GIF images making up the parenthesis) that looks like

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

As mentioned above, other topics can easily be integrated into LAVA. Another possible (more extensive) modification would be to have LAVA do some student evaluation, perhaps by recording the student responses to "summary quizzes" and reporting these results to the instructor. Students could "log in" and pick up where they left off as they work toward completing assignments.

References

[1] *The Linear Algebra Curriculum Study Group Recommendations for the First Course in Linear Algebra,* The College Mathematics Journal, **24**, No. 1, January 1993.

[2] S. Leon, E. Herman and R. Faulkenberry, editors, *ATLAST Computer Exercises for Linear Algebra,* Prentice-Hall, 1996.