Mathematical Models and Reality

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Abstract

This paper examines the nature and function of mathematical models, using illustrations from cosmology, space geometry and atomic physics. Mathematical models enable us to make precise calculations and predictions; they serve as analogies and conceptual frameworks that lead to new discoveries; and they bridge the gap between appearance and reality. Their success implies that the universe has a mathematical structure. However, one must be careful not to confuse models of reality with reality itself. A variety of models can represent the same data; any model can be given different physical interpretations. The choice of a model and its interpretation depends largely on one's worldview

1. Introduction

Mathematical models play a large role in science. In this paper we shall investigate a number of ways in which models have been used. We shall illustrate these by examining a number of historical examples, taken from cosmology, space geometry, and atomic theory. We shall examine the strengths and weakness of mathematical models.

2. What is a "Model"?

What do we mean by a mathematical "model"? Webster's Ninth New Collegiate Dictionary (1989) gives a dozen or so definitions of "model". Among these, the most pertinent for our purposes are the following:

4. *a miniature representation of something.*

11. *a description or analogy used to help visualize something (as an atom) that cannot be directly observed.*

12. *a system of postulates, data, and inferences presented as a mathematical description of an entity or state of affairs.*

The Oxford American Dictionary and Language Guide (1999) gives a few further definitions, including:

2. *a simplified (often mathematical) description of a system, etc., to assist calculations and predictions.*

From these definitions we shall tentatively take a model to be a simplified, often mathematical, representation or analogy of some aspect of reality, for the purposes of description and/or calculation.

Models in science come in a wide variety of forms. Ian Barbour (1974:29), in his extensive discussion of models, lists four types of models:

1. *Material* models. These are simplified, scaled-down (or up) miniatures (e.g., wind tunnels, hydrodynamic models) or analogue models (e.g., an electric circuit has the same behavior as a mechanical system of springs & dampers). They are useful when it is too difficult to experiment on the actual system or when the mathematical equations are unknown or too hard to solve.

2. *Mathematical* models. These are sets of equations, using symbolic representations of quantitative variables in simplified physical or social systems (e.g., population growth, economics). Their chief function is prediction.

3. *Logical* models. A logical model is a particular set of entities satisfying a given set of axioms. For example, points and lines in geometry is a logical model for Euclid's axioms; arithmetic is a model for Peano's axioms. However, any set of axioms also has other, unintended models. These models serve as illustrations or interpretations of the initial system of abstract axioms.

4. *Theoretical* models. These are imaginative mental constructs invented to account for observed physical phenomena. They are usually mechanisms or processes, postulated by analogy with familiar mechanisms or processes (e.g., the billiard ball model for gas, Maxwell's mechanical model for electro-magnetic forces, etc.). Their chief use is to help us understand the world rather than merely to make predictions. They are symbolic representations of physical systems, aiming to represent the underlying structure of the world. A theoretical model is a framework of ideas and concepts from which we interpret our observations and experimental results. In its highest form, a physical model is expressed as a set of natural laws (e.g. Newton's laws of motion). The physical theory should be based on a minimum number of physical assumptions, and, ideally, must broaden our understanding of the physical phenomenon.

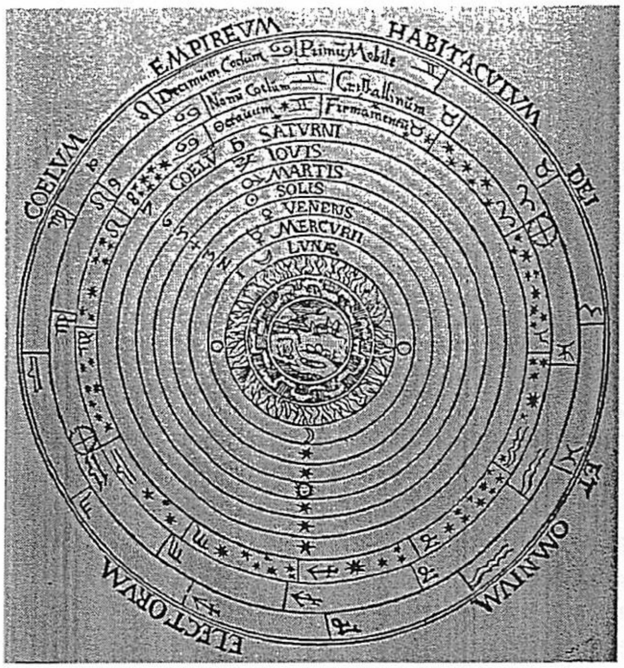
A mathematical model is usually a built-in element of a theoretical model. The physical theory interprets the mathematical model, including its assumptions and constraints. The mathematical model is needed to quantify the physical model, thus enabling the theoretical model to make precise predictions and applications.

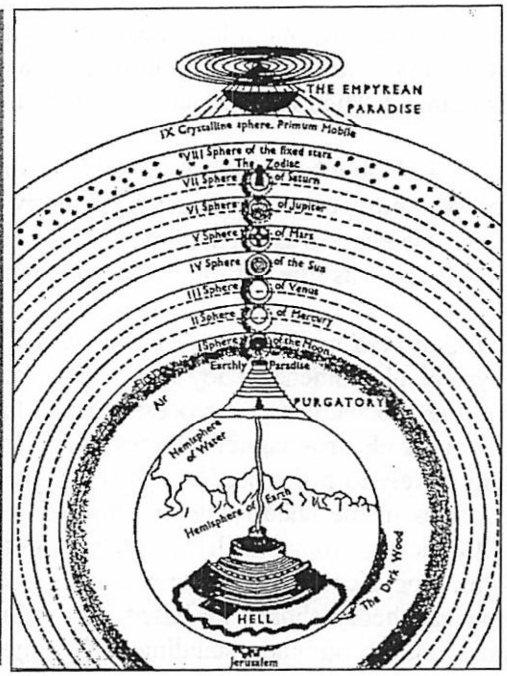
3. Models in Historical Cosmology

We shall consider first the role of mathematical models in cosmology. A prime goal here is to give a simplified representation of the physical universe in terms of a few basic theoretical presuppositions. Ancient Greek cosmology—particularly that of Plato and Aristotle—was generally geocentric. The celestial objects circled around a fixed Earth. Uniform circular motion was thought to reflect the perfection of the heavens. The earliest theoretical models consisted of the planets embedded in glass shells. The Prime Mover, situated beyond the outer shell, set the outer shell in motion. From there, motion was mechanically transmitted inwards to the other shells.

Many ancient cosmological models reflected also theological truths. In medieval cosmology heaven, for example, was placed beyond the outer sphere (see Figure 1A). Dante's theological/physical model (Figure 1 B) included Hell, with its various levels, deep inside the Earth. Such models involved a presentation of theological truths and did not aim at an accurate quantitative description.

Plato already noted that these simple models did not quite represent the actual motion of the planets, which differed significantly from uniform circular motion. Hence Plato set his students the task of devising improved mathematical models that would better 'save the phenomena.' The aim was to give an accurate cosmological description based upon uniform circular motions in the celestial realm. Aristotle tried to solve the problem by using 55 glass intermediary spheres but this sophisticated model still fell short of the observations.





A B

Figure 1. ***Medieval Cosmological Models***

Ptolemy's Epicycles and Instrumentalism

The problem was eventually solved by Claudius Ptolemy, around 150 A.D. He invented a number of novel geometrical devices. These included the *epicycle* (a small circle superimposed upon a larger circle, called the *deferent*), the *eccentric* (a device making the center of the circle rotate off-center about the Earth), and the *equant* (another off-center point from which speeds were calculated, in order to retain uniform speeds. These concepts are depicted in Figure 2. The resulting geometric model worked very well. It yielded results that closely approximated the observed motions, thus enabling astronomers to predict future planetary positions. However, in the case of some planets it was found necessary to add another layer of epicycles, smaller epicycles moving about larger ones, to adequately describe the observed motions. The complete Ptolemaic system consisted of 40 epicycles.

Although Ptolemy's model gave useful predictions, it was a purely mathematical model with no physical underpinnings, other than satisfying the classical demand for a combination of uniform, circular motions. Ptolemy defended his mathematical model by adopting an anti-realist (i.e., instrumentalist) view of mathematical models. He presented his model as just a useful fiction for practical prediction. His prime criteria were (1) accuracy in saving the appearances and (2) maximum simplicity. His approach of adding layers of epicycles corresponds to the modern application of a Fourier series.

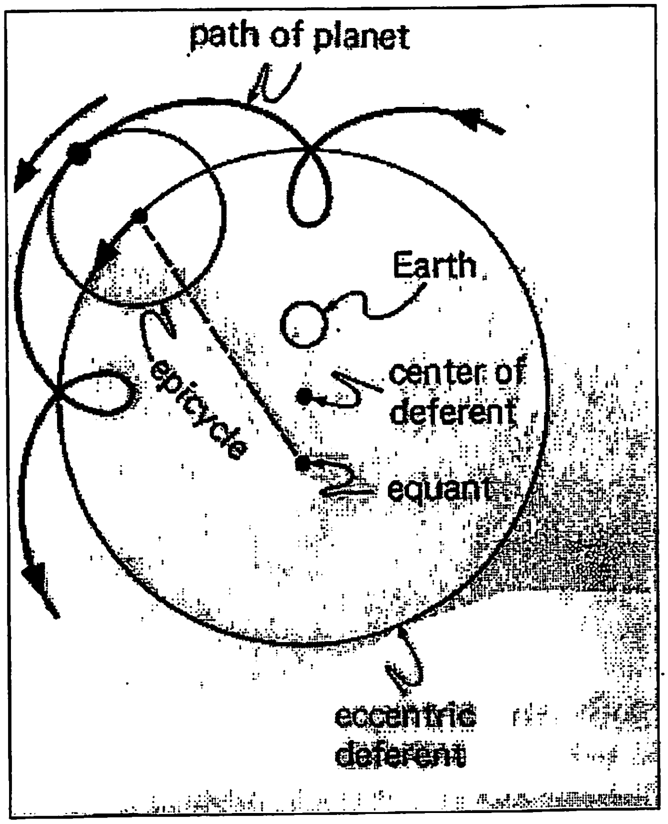


Figure 2. ***Epicyclic Theory.*** A planet revolves about a small circle, an *epicycle*, which in turn revolves about a larger circle, the *deferent*. The deferent is *eccentric* when its center is not at the Earth. The *equant* is a non-central point about which the epicycle moves at a constant angular rate

This view of scientific theorizing was quite different from the rival, "realist" position that had been defended by Aristotle, who believed that theories should do more than merely fit the observations. Aristotle insisted that models should be in accord with the true nature of things. Thus his followers rejected Ptolemy's system since it contradicted the principles of Aristotle's physics.

Copernicus and the question of the Earth's motion

The proper function of cosmological models was much discussed with the advent of Copernicus' (1543) heliocentric model. Copernicus' model was a purely geometric one, with no basis in physics. It kept to pure circles (with epicycles) and was no simpler than Ptolemy's model. However, it did explain the hitherto puzzling retrograde motion of planets. This was now seen as only an apparent effect, due to the relative motions of the Earth and outer planets about the Sun. Although Copernicus believed his model to be a true representation of reality, the forward to his book (written by the Lutheran theologian Osiander) presented it as merely a useful calculating device.

A few years later (in 1588) Tycho Brahe presented an alternative model in which all the planets revolved around the Sun, which in turn moved about a fixed Earth. This model was observationally equivalent to that of Copernicus. How, then, was one to choose between them? Tycho held that, since his model accorded with the geocentric cosmology of the Bible, it was theologically preferable.

The Copernican model was not widely accepted as a true depiction of reality until about 1650, when a physical basis was finally found for it, in the form of Descartes' theory of vortices. Descartes' theory was soon falsified by Newton (in 1689). Newtonian mechanics, on the basis of its inertial frames of reference, supported a universe in which *both* the Earth and the Sun were in absolute motion. It was Newton, above all, who undermined geocentric models of the universe.

The notion of absolute motion was again transformed with the advent of general relativity. Lynden-Bell (1995) has shown that, in general relativity, the universe rotating about a fixed Earth would produce Coriolis and centrifugal forces, the bulge at the Earth's equator, and all other phenomena generally adduced to prove that the Earth is rotating. In that case, how do we choose between Copernicus and Tycho? At this point one might well ask, what does it mean it say that the universe--or the Earth--is at rest? At rest with respect to *what*? The question implies that there is some fixed standard of rest outside the universe. But how would we determine what that would be? And with respect to what is *it* at rest? The standard of absolute rest is largely a matter of definition, which depends on extra-scientific considerations. Ultimately it is thus a question of philosophical and theological preference.

In short, one is generally not satisfied with a model that merely saves the phenomena: it should preferably do so in terms of plausible physical principles. The plausibility of physical principles in turn depends very much on our worldview. One important function of mathematical models (such as Tycho's) is to provide a precise mathematical connection between one's worldview and one's observations. Such connections serve to increase the plausibility of that worldview.

4. Models of Space

Next we consider models of space. Until quite recently, Euclidean geometry was widely considered to be the true model of physical space. The discovery of non-euclidean geometries raised the questions: which is the true geometry, and what is the status of the other geometries?

Poincare's Geometric Models

Henri Poincare argued that geometry is largely a matter of convention. He asserted that we can have *either* (1) simple physics (e.g., light travels in straight lines) and complicated geometry (e.g., curved space) *or* (2) simple geometry (e.g., Euclidean space) and complicated physics (e.g., light travels in curved paths).

Mathematicians often use the notion of a model in the sense of a mathematical representation designed to study another mathematical structure. Thus Poincare, for example, proved the consistency of non-euclidean geometry by constructing a model which embedded it within Euclidean geometry. The consistency of non-Euclidean geometry then follows from the consistency of Euclidean geometry. Poincare's model for plane hyperbolic geometry is well-known. The infinite hyperbolic plane is represented by a finite circular disk in the Euclidean plane. Lines in the hyperbolic plane are represented by arcs of circles perpendicular to the boundary of the disk. M.C. Escher's artistic depiction of this hyperbolic model is shown in Figure 3.

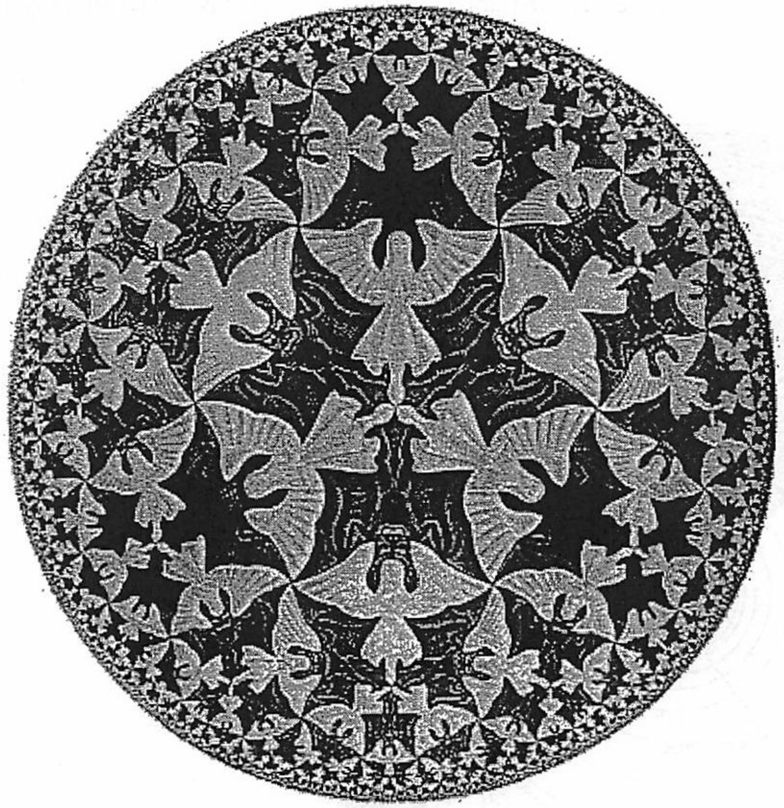


Figure 3. A model for the hyperbolic plane. *Heaven and Hell* by M.C. Escher.

Not quite as well-known are Poincare's models of 3-d non-Euclidean space. Imagine space to be filled with small metal balls, whose size is proportional to the temperature T of space. Euclidean space is represented by a constant temperature, so that the spheres are uniformly the same size throughout space. A model for infinite hyperbolic space can be constructed by taking a finite Euclidean sphere of radius R with a temperature variation of $T = k\ (R^2 - r^2)$, where k is a constant of proportionality and r is the distance of a ball from the center. The metal balls then shrink to nothing as they approach the edge (this is the 3-d version of Figure 3). For a moving object, its speed likewise diminishes as it approaches the edge, so it never quite reaches the edge.

Similarly, we can model (finite) elliptical space of radius R in the same Euclidean sphere by letting the temperature vary as $T = k/(R^2 - r^2)$. Now the spheres grow infinitely large as they approach the edge, thus re-appearing on the opposite side.

Such modeling of non-euclidean geometries within the more familiar euclidean space helps us to visualize the properties of such novel geometries. This illustrates a further function of mathematical models: to represent various aspects of reality that are otherwise hard to visualize. Mathematical models help to translate novel conceptual geometries into the more common Euclidean space of our everyday experiences.

Of Earths Inverted and Flattened

Closely related to these geometrical models are some unusual conceptions of the universe. For example, Fritz Braun (1973) asserts, based on his interpretation of biblical texts, that the Earth should be inverted. The Earth's surface is the *inside* of a hollow sphere enclosing the Sun, Moon, and stars. Heaven is at the center of the inverted universe, thus making this model literally theocentric (see Figure 4).

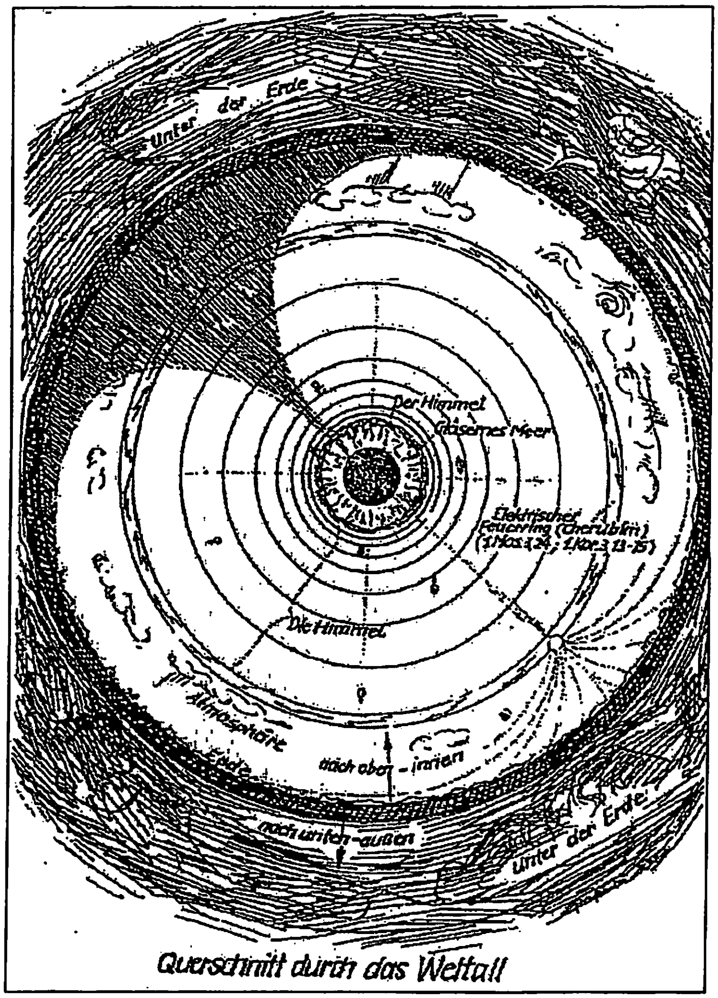


Figure 4. ***Braun's Inverted Universe.*** Note that heaven is at the center, surrounded by the glassy sea, the planets, Sun and clouds.

At first sight this model seems obviously false. One might think, for example, that this model entails that we should be able to see across the hollow sphere to the other side of the Earth. Indeed, in 1933 German promoters of the hollow Earth theory tried to prove their theory by means of rockets. They reasoned that a rocket, fired straight up, should hit the opposite side of the Earth. Various rockets were fired but, unfortunately, they all malfunctioned and the test was eventually abandoned.

However, this model is not that easily dispensed with. It can be devised so that disproof is impossible. The above tests take for granted that the normal laws of physics hold. In particular, light is expected to travel in roughly straight lines and rockets, in the absence of forces, are expected to move at a constant velocity. But what if this is no longer the case?

The hollow Earth model can be derived from the more usual picture of the universe via a simple mathematical transformation called a "geometric inversion". The procedure is very simple. For each point in the universe, measure its distance r from the center of the Earth and move the point along the center-to-point line to a new distance 1/r. The result of this operation is that all objects originally outside the Earth (e.g., mountains, houses, clouds and stars) are now inside, and vice versa (see Figure 5). Inversion is a *conformal* transformation, which means that local shapes are preserved.

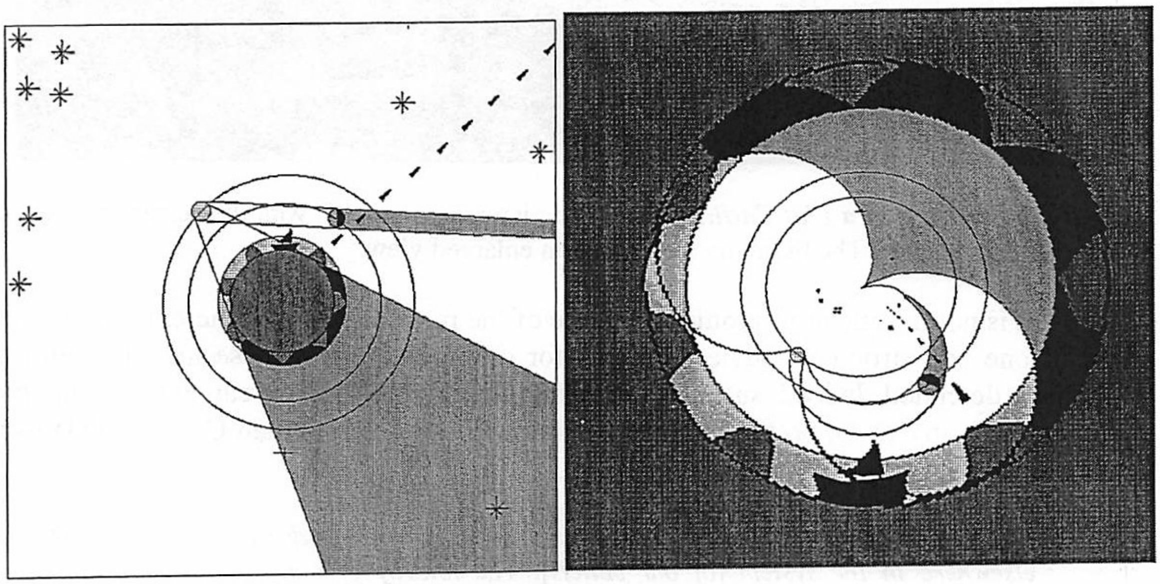


Figure 5. ***A Simple Model of the Universe and Its Inverse.*** The second figure is the result of inverting the first figure, taking the earth's center as the center of inversion. For ease of comparison, the first figure has been flipped horizontally. Note the curved light rays and the diminishing size of the rocket as it recedes from the earth.

The laws of physics are also inverted, with consequences that may seem strange for those accustomed to thinking in terms of the more conventional universe. For example, light now travels in circular arcs. Also, a rocket launched from the Earth to outer - or, rather, now "inner" - space will shrink and slow down as it approaches the central heaven, never quite reaching it (see Figure 5).

Consequently, Braun's inverted universe is observationally indistinguishable from more conventional models of the universe. Yet, although the two models are empirically identical, they involve quite different ways of viewing reality. Braun's model reflects his theological beliefs. Again, the mathematical model functions here to connect a particular worldview with observations, thus making that worldview more viable.

Note that, if we were to take a point on the Earths' surface as the center of inversion then we would get a flat Earth (i.e., this is the *stereographic* projection of geography). As you travel to the edge you become infinitely large at the edge, so that you re-appear at the right (see Figure 6). Again, this model is observationally undisprovable.

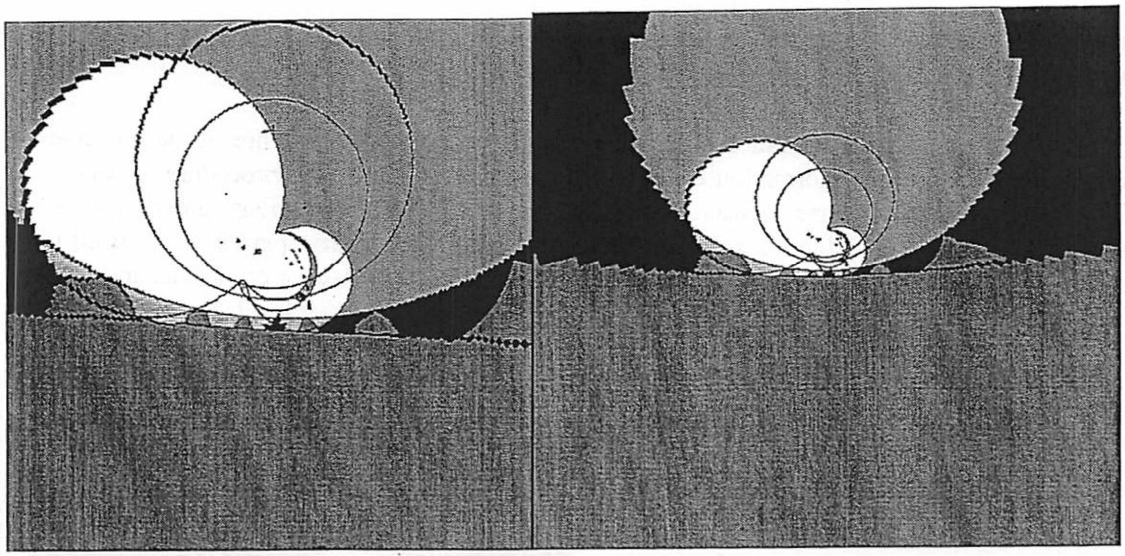


Figure 6. ***Inverting to a Flat Earth.*** An inverted picture of Figure 5, with the center of inversion on the earth's surface. The figure on the right is an enlarged view.

It is not my intent to promote any one of the models discussed here. I merely note that, if one had strong worldview reasons for doing so, any of these models could be rationally defended. Indeed, such extreme examples suggest that we can always construct a model with any given preferred characteristics. Willard Van Orman Quine (1953) went so far as to claim,

> *"Any statement can be held true come what may, if we make drastic enough adjustments elsewhere in the system [of our beliefs]. The totality of our so-called knowledge or beliefs, from the most casual matters of geography and history to the profoundest laws of atomic physics or even of pure mathematics and logic, is a man-made fabric which impinges on experience only along the edges."*

From this we get the *Duhem-Quine Thesis*, which asserts that we can always construct a model with any given feature, if we make drastic enough changes elsewhere in the system. Along such lines the cosmologist Frank Tipler (1984: 873) writes:

> *"It is universally thought that it is impossible to construct a falsifiable theory which is consistent with the thousands of observations indicating an age of billions of years, but which holds that the Universe is only a few thousand years old. I consider such a view to be a slur on the ingenuity of theoretical physicists: we can construct a falsifiable theory with any characteristics you care to name."*

Nevertheless, although such claims may be true, there is always a price to pay. This price comes in terms of the, often drastic, secondary changes needed to link the preferred characteristics to the observational data. For example, in the inverted model of the flat Earth, the price tag is that light no longer travels in straight lines, that objects change size as they move, etc. These, in turn, may compel us to reconsider how to define and measure "flatness". It is not easy to "sell" a model that requires a wholesale modification of one's intuitive common sense, even when one acknowledges that such common sense may itself be largely the result of having bought into some previous model of reality.

5. Models of Atoms

Finally, we consider some models of atoms. What is matter really like? My desk seems to be made of solid material. However, when we consider models of matter, we find quite a range of historical representations. John Dalton's (1803) model was just one of solid, uniform material. In 1904 J.J. Thompson replaced that with his plum-pudding model, where negative charges (i.e., electrons) were placed in the uniform, positively charged, matter like so many plums or raisons in a pudding (see Figure 7). Soon thereafter Ernest Rutherford (1911) asserted that the atom was mostly empty space. Niels Bohr (1913) added the restriction that electrons can occupy only certain, discrete orbits. These latter models draw upon the notion of action at a distance (already introduced by Newton and Faraday), which is explained in terms of fields. But are electro-magnetic fields actual physical phenomena or just abstract mathematical concepts?

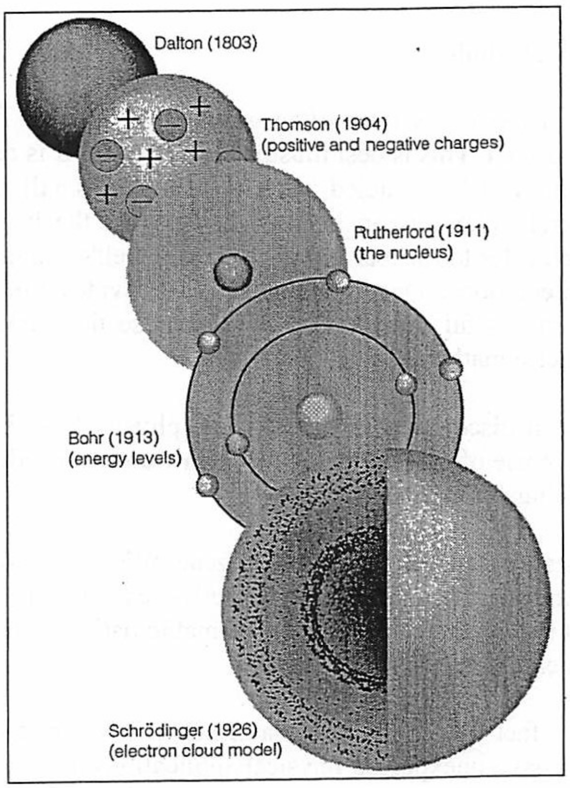


Figure 7. ***Various Models of the Atom***

Finally, Edwin Schrödinger replaced these definite orbits with the fuzzy electron cloud model of quantum mechanics. Quantum mechanics raises many more questions about models. Quantum mechanics uses wave-functions. But what do the quantum wave-functions represent? Schrödinger initially viewed them as physical vibrations in an electromagnetic field. In such terms the electron was no longer a particle but became a collection of wave disturbances. Against such a realist interpretation of the wave function were the facts that they could have imaginary (i.e., involving $\sqrt{-1}$) components and, for anything more complicated than hydrogen, consisted of more than 3 dimensions. On such grounds Max Born argued that they represent, not reality, but only our *knowledge* of the state of a physical object; hence they should be interpreted merely as probabilities. In quantum mechanics, although there is general agreement regarding the *mathematical* model, there is much contention regarding the *physical interpretation* of this model.

Reality becomes even more nebulous when we consider the never-ending invention of new particles on the sub-atomic level: quarks, leptons, gluons, anti-particles, etc. Do such particles really exist or are they just mathematical models for observed energy relations? Thomas Hillen (2003) argues that "quark" simply means a model of an object that is characterized by certain quantum numbers, like spin 1/2, Baryon number 1/3, Lepton number 0 and charge +2/3 or -2/3. A "particle" is usually represented by a solution of a quantum dynamical Schrödinger equation, which is a mathematical model for electromagnetic interactions. Hillen asserts, "A question like 'Does this particle exist?' must be understood as, 'Does this model describe some experiment that cannot be described without this model?'"

Which desk, then, is the real desk? The solid desk I actually experience? A shadowy shape consisting of tiny particles bouncing about in mostly empty space? Or, a mathematical abstraction of complex wave-functions?

6. The Amazing Success of Mathematical Models

One remarkable feature of mathematical models is their often astounding success. Frequently they work much better than might be expected. This is best illustrated in physics. It is remarkable that a wide range of physical phenomena can be modeled in terms of a very small number of physical principles. For example, general relativity can be used to describe the behaviour of objects ranging from billiard balls and bicycles to rockets and planets. Maxwell's equations allow us to describe all electro-magnetic interactions. Quantum mechanics provides the basis for chemistry. Physics has been a highly successful science primarily because the basic physical principles can be readily modeled by precise mathematical equations.

According to Max Born, "all great discoveries in experimental physics have been made due to the intuition of men who made free use of models which for them were not products of the imagination but representations of real things" (Barbour: 47).

Why are mathematical models so successful? In 1960 Eugene Wigner, a Nobel-prize winner in physics, gave a famous lecture on "the unreasonable effectiveness of mathematics in the natural sciences". He concluded that the amazing applicability of mathematics to the physical world is a mysterious, undeserved and inexplicable gift.

Part of the mystery is due to the fact that sometimes mathematics developed for purely mathematical purposes later turns out to have unexpected physical applications. For example, in 1609 Johannes Kepler found that planetary orbits can best be described in terms of ellipses, mathematical curves that had been studied two thousand years earlier by Greek mathematicians. Steven Weinberg, another Nobel-prize winner in physics, remarks

> *"physicists generally find the ability of mathematicians to anticipate the mathematics needed in the theories of physics quite uncanny. It is as if Neil Armstrong on 1969 when he first set foot on the surface of the moon had found in the lunar dust the footprints of Jules Verne"* (1992: 157)

Philosopher Mark Steiner (1998) notes that Wigner's "mystery" is open to various objections. For example, Wigner ignores the failures, where appropriate mathematical descriptions could not be found, as well as those mathematical concepts that have never found any applications. Moreover, many mathematical models turn out to be inadequate. Consider, for example, Kepler's 1596 model of the solar system. Kepler found that the spacing between the planets closely approximated the spacing of the 5 Platonic solids when fitted inside each other in a specific order (Figure 8). This model had no physical basis at all, being based purely on mathematical considerations. It was falsified with the discovery of the 6th planet, for which there could be no corresponding platonic solid. Physicist S. Sternberg (1969) sees a similarity here with the introduction of pure mathematics into modern particle physics:

> *"Kepler's principal goal was to explain the relationship between the existence of five planets (and their motions) and the five regular solids. It is customary to sneer at Kepler for this. It is instructive to compare this with the current attempts to "explain" the zoology of elementary particles in terms of irreducible representations of Lie groups"*

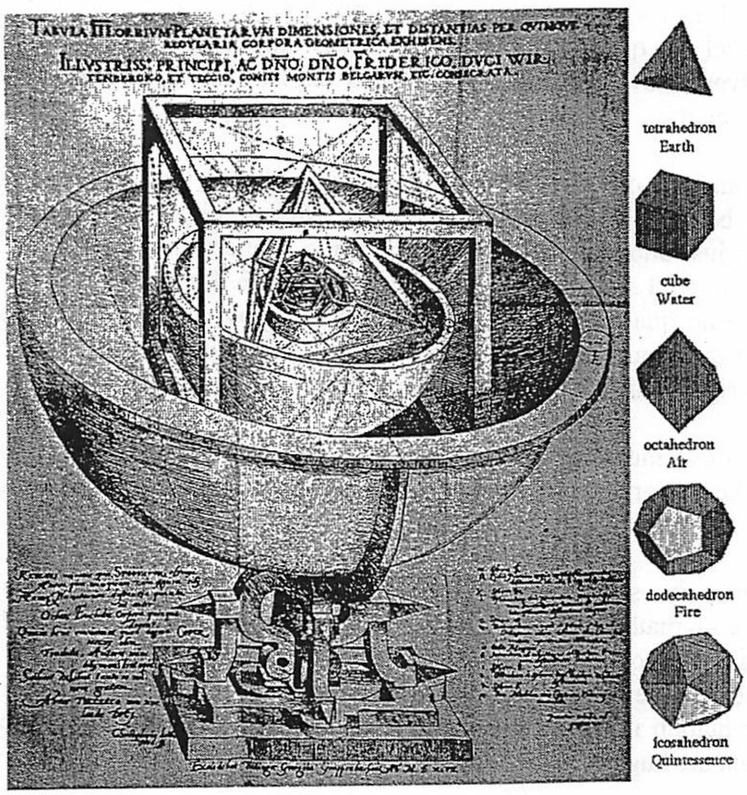


Figure 8. ***Kepler's Platonic Solid Model of the Solar System.***

Nevertheless, Steiner believes that Wigner is on to something. He contends that the applicability of mathematics concerns not just a few isolated successes in physics. Rather, it pertains to the much broader applicability of mathematics as a global research strategy. Physicists, from Kepler and Galileo onwards, have been gripped by the conviction that mathematics is the ultimate language of the universe. Physicists probe nature with an eye for mathematical structures and analogies in nature.

But such a mathematical research strategy for making discoveries is essentially an *anthropocentric* (i.e., man-centered) strategy. It presumes that humans have a special place in nature. This is because mathematics relies on human standards such as simplicity, elegance, beauty and convenience. Anthropocentrism is most blatant in those cases where even the *notation* of mathematics plays a role in scientific discovery.

Steiner gives the example of Paul Dirac's discovery of the positron. In 1930 Dirac applied quantum mechanics and special relativity to electrons. He ended up with a quadratic polynomial that had to be factored. When real and complex numbers did not work, Dirac introduced higher-dimensional number-like objects (4 dimensional matrices). This factoring yielded several extra solutions, in addition to that corresponding to the electron. One of these solutions implied the existence of a particle identical to the electron but with a positive charge. Two years later, the existence of such particles--called *positrons*--was confirmed experimentally. Thus a mere mathematical trick, invented for computational convenience, resulted in a major physical discovery. Remarkably, the mathematical method Dirac applied (known as *Clifford algebra*) had been developed already in the 1800's for entirely different, purely mathematical, purposes.

Quantum mechanics involves further mysteries. Most quantum physicists hold that we cannot adequately picture or model the quantum realm. This leaves only the *formalisms* of the mathematical equations themselves. How can mere formalisms explain the great success of quantum mechanics? Steiner comments:

> "The success of the formalism of quantum mechanics in predicting the properties of helium should have no bearing on its probable success with uranium...to say that a connection is "formal" is just another way of saying that the connection is mediated by nothing more than notation. And a connection mediated by notation, I have been arguing, is anthropocentric...I have no quarrel with a physicist who is happy with the status quo, and works with quantum mechanics as a mere formalism. My only claim is that such a "happy" physicist has no right to be a naturalist" (Steiner 1998: 145-6)

So the philosophical problem is not so much the applicability of mathematics to our *descriptions* of physical reality but, rather, the major role of *human* mathematics in the *discovery* of new phenomena.

Steiner concludes that our universe appears to be intellectually "user friendly" to human. His examples of the amazing use of mathematics in scientific discovery, as well as in scientific description, argue strongly against the notion that mathematics is just a human invention. This favours the realist view that mathematics exists objectively, in some ideal, non-physical realm, with both the physical world and our human minds somehow reflecting aspects of that mathematical realm. These features would explain the huge success of our mathematical models.

7. The Fallacy of Misplaced Concreteness

However, we must be careful not to over-rate mathematical models. For example, according to Roger Penrose, the concrete world of physical reality emerges mysteriously out of the ideal world of mathematics. Penrose views the mathematical world as the primary, real world; the other two worlds of our consciousness and physical things being mere shadows of it (1994: 417). Similarly, the Oxford chemist Peter Atkins postulates that the ultimate reality is mathematics and that the universe originates from a mathematical set of points (Atkins 1994: 128). Other scientists have made similar remarks.

The views of Penrose and Atkins illustrate a peril facing scientists, particularly mathematical physicists. This danger is what Keith Ward calls "the fallacy of misplaced concreteness" (Ward 1996: 28). One is so impressed by the beauty and predictive power of one's mathematical model of reality that one comes to see one's model as the true reality. The fallacy is to mistake the *abstract* model for the *concrete* reality. The true reality then becomes an abstraction, such as Hamiltonian vector fields in multi-dimensional phase space, whereas one's actual concrete experiences, upon which the model is ultimately based, are then relegated to the realm of mere subjective illusion.

This fallacy occurs in its strongest form in materialist reductionism, which attempts to explain all of reality--including our minds--in terms of purely physical laws. For example, the eminent, Nobel prize-winning biologist Sir Francis Crick opens his book *The Astonishing Hypothesis* with:

> "The Astonishing Hypothesis is that "You", your joys and your sorrows, your memories and your ambitions, your sense of personal identity and free will, are in fact no more than the behaviour of a vast assembly of nerve cells…" (1994: 3)

According to Sir Francis, the real "you" is just an illusion caused by brain processes and the ultimate reality consists only of atoms and their interactions. This curt dismissal of our "self" as a real entity contradicts our strongest, most intimate, conscious experiences.

On the contrary, one must never forget the proper limits of models. Models of the universe function like maps of landscapes. A map is just an *abstract representation* of the landscape and, as such, should never be mistaken for the real thing. The mathematical world consists of timeless, abstract truths. It is an ideal realm of pure thought. The physical world, on the other hand, consists of contingent, temporal, concrete particulars.

How can abstract universals produce concrete, contingent, physical facts? Abstractions and universals are of themselves inert. Hence, the move from mathematical abstraction to the physical world seems to require an active, necessary being. Stephen Hawking, after trying to show that the world is self-contained, needing no Creator, nevertheless concludes his book *A Brief History of Time* with the words,

> "Even if there is just one possible unified theory, it is just a set of rules and equations. What is it that breathes fire into the equations and makes a universe for them to describe? The usual approach of science of constructing a mathematical model cannot answer the questions of why there should be a universe for the model to describe. Why does the universe go to all the bother of existing? Is the unified theory so compelling that it brings about its own existence? Or does it need a creator, and, if so, does he have any other effect on the universe? and who created him?" (1988: 174)

This underscores the crucial distinction between concrete physical reality and a model's abstract presentation of that reality.

There is a closely related danger. Physical, theoretical models, such as quantum mechanics or superstring theory, are generally quantified by means of mathematical models. The danger is that abstract mathematical concepts, introduced during the development of a mathematical model, become seen as physical concepts. This may make physical reality seem much stranger than it actually need be.

One example of an abstract mathematical concept thrust into the realm of physics is that of higher dimensional space, such as the 10 (or more) dimensions of string theory, the multi-dimensional phase space of Hamiltonian mechanics or the infinite Hilbert space of quantum mechanics. Such spaces may be very convenient mathematical concepts but should not be mistaken for physical reality.

Conclusions

From our discussion it is evident that mathematical models serve a variety of purposes. Our main conclusions can be summarized as follows:

1. A prime pragmatic function of such models is to enable calculations and predictions of physical phenomena. Mathematical models are useful also in representing aspects of reality that are hard to visualize. In that respect they function as metaphors and analogies. Models serve as conceptual frameworks that can lead to important physical discoveries.

2. The astounding success of mathematical models suggests that the universe has an underlying mathematical structure that is discernable to humans. This, in turn, suggests that mathematical models reflect, to at least a limited degree, a deeper reality that goes beyond mere appearances.

3. On the other hand, we noted that a variety of different mathematical models can account for the same appearances (e.g., the models of Ptolemy, Copernicus and Tycho). Also, a given mathematical model can be given many different physical interpretations (e.g., quantum mechanics). This implies that the construction of models involves a substantial element of creative imagination.

4. This raises the question as to how models help to relate appearances to reality. Our conception of reality (i.e., metaphysics) and how we acquire knowledge of it (i.e., epistemology) largely depend on our worldview, which is based on our most basic beliefs (i.e., presuppositions). Our choice of models is generally made on the basis of worldview presuppositions. These presuppositions determine what we consider to be real and what we dismiss as only apparent. We saw that mathematical models provide useful bridges connecting our worldview with our experiences. Such connections serve to increase the plausibility of our worldview.

Worldviews stressing empiricism (i.e., the notion that all valid knowledge comes through our senses) will tend to see models as little more than black boxes, having data as input and predictions as output, with little concern for the truthfulness of the contents of the black box. But this makes the success of models seem miraculous. Some of these more philosophical issues related to models and reality are discussed in more detail by G.Y. Nieuwland (2000).

5. Finally, concerning the fallacy of misplaced concreteness, we noted that models are merely abstract representations of reality, not reality itself. Similarly, abstract mathematical concepts in models should not be mistaken for physical concepts.

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