# The Inverse Problem: Christianity through a Mathematical Lens

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## 1 Introduction

Mathematicians enjoy thinking about problems. When posed with a new idea we experiment with special cases, we look for patterns, we conjecture, we generalize, we prove or disprove our conjectures–and then we generalize again. But we are never finished there. The fun part still remains–what if we look at the situation in reverse? How do our experiments behave? Are there any new patterns? Can we generalize in a different way to learn more about the problem? (See Figure 1)

Now I know that this process is familiar. Every mathematician has experimented with concepts in this way. In fact we teach our students to try to think about a problem from many directions–it is embedded within the standard undergraduate curriculum! We teach our students to differentiate functions and then we turn the problem around and ask them to integrate. We discuss a mathematical statement and then we consider the converse. We investigate a three-dimensional image of a surface by looking at level curves in the two-dimensional realm. Then we look at a collection of level curves and try to visualize what three-dimensional image might have these slices. Of course, you know the next step is to generalize the process to hypersurfaces in four-dimensional space with "shadows" in a three-dimensional setting.

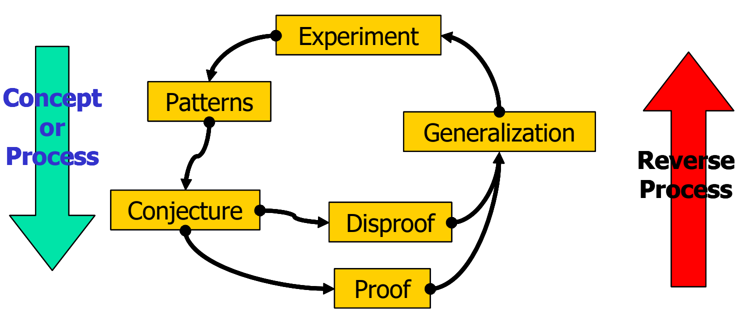



Figure 1: Mathematical Investigation

## 1.1 Inverse Problems

All of these examples fit to a general category of problems called "inverse problems." To be a part of this category of problems a few general principles must be satisfied. First, every inverse problem contains a partnered pair of problems. One problem is called the "direct" problem, the partner is called the "inverse" problem. For these direct and inverse problems to be genuine partners, each must in some way reverse the process of the other. In the above situation with the three-dimensional surface, either you knew the level curves and tried to determine the shape of the object OR you knew the shape of the object and tried to determine the appearance of the level curves. Second, in most cases, one of the paired problems is more difficult to solve. Perspective and experience determines which problem is the harder of the two. The "inverse" label is usually given to the more difficult problem of each pair. Finally, if one of the paired partners has a unique solution, there is no guarantee that the other partner will also have a single or even a *finite* quantity of answers. Often "the data for the inverse problems lack[s] essential information necessary to uniquely reconstruct the object." [3, page 5]

## 1.2 Examples of Inverse Problems

There are innumerable inverse problems within mathematical and nonmathematical settings. Here are a few additional examples.

- The television game show "Jeopardy" is set up to be in inverse form. Contestants are given the answer and must provide the question.
- Experienced drivers know that driving a car in reverse is more difficult than driving the car forward–especially when pulling a trailer. So the inverse problem is driving in reverse without causing an accident.
- In Linear Algebra students invert the simple process of matrix multiplication to decompose a coefficient matrix into a product of matrices. Matrix factors are selected to have special forms (e.g., $LU$ or $QR$) to improve computational efficiency in solving a system of equations.
- In cryptography, trap-door functions are crucial to the security of an encryption method. Encryption and decryption are easy for users with special information–i.e., the trap door–but extremely difficult for someone without the special information. Processes which are practically but not proven irreversible include multiplication of large primes, exponentiation modulo large moduli, and multiplication in an elliptic curve group. In these, the inverse problem is factoring in the first case and solving a discrete logarithm problem in the latter two cases.

### 1.3 Mathematics and a Christian Perspective: an Inverse Problem?

Mathematics and theology are very different spheres of study. Understanding and explaining the relationship between the two in a meaningful way is particularly difficult–Christian mathematicians have struggled with this for a number of years. I believe that by recognizing that two approaches to the investigation of the connection form an inverse relationship helps to make the problem manageable. In my experience, acknowledging that a problem is difficult allows me to be satisfied with understanding very small pieces and making slow progress toward a complete and satisfactory solution. My thesis is that to use mathematical thinking to understand the concepts of theological principles is the direct problem; that is, the easy problem. However, using theological thinking to influence understanding in mathematics is the inverse problem; this problem is much more difficult to address.[1]

This separation into two inverse problems is not completely new for Christian mathematicians to consider. In fact, J. Mann of Wheaton College recognized the nature of the two directions in 1985. He writes, "I propose that it may be more fruitful to reverse the usual question ... and ask: How does being a mathematician affect my view of Christianity?" His approach in solving the direct problem focuses more on general mathematical principles such as precision, abstraction, and axiomatization rather than on concrete connection with content area mathematics. [4] Others who have discussed various ways to solve the same question include P. Bialek with a focus on paradoxes [2] and A. Reiter Ahlin who writes in preface to her parable *Good News for Curved Beings*, "Mathematical images are the deepest, biggest, strongest images I know. So I'd like to use them to meditate on spiritual truths." [1]

There are several reasons why being aware of component pieces of the issue of integration of faith and mathematics is significant. First, the separation gives validity to a "divide and conquer" approach to the problem. Where one scholar might have gifts which make one aspect of the problem accessible, a second will be better equipped to work on another. Second, awareness of the diverse structure of the problem will help the larger community to classify progress on the broad integration of faith and mathematics question. Knowing how to position new scholarship helps to broaden understanding and can open avenues for collaboration. Finally, better understanding of one aspect of the problem can open new doors for investigation of another area of the problem.

The development of my own thinking on the direct problem began several years ago after read an article called, "Higher Dimensions in the Writings of C. S. Lewis, " by David Neuhouser of Taylor University. [5] This article opened my eyes to the way that mathematical thinking can effectively model complex theological concepts. For example, Neuhouser highlights writings of Lewis that uses the same type of reduction of dimension argument as the one multivariable calculus students use to visualize hypersurfaces. In these writings, Lewis explains that Christ's post-resurrection body was able to enter and leave

[1] As noted above, the catagorization of "inverse" problem is rooted in the experience of the observer. Those with philosophical natures might disagree with my assessment.

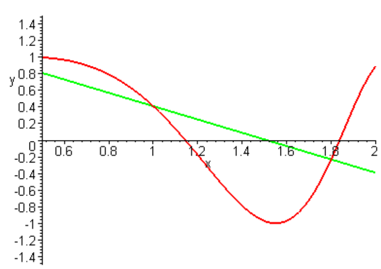
the upper room without passing through a locked door because the disciples could only see a three-dimensional shadow of Christ's higher-dimensional body.

After reading this article, I started looking for additional ways that mathematics could effectively model theology. I found mathematical concepts in sermons, in scripture readings, and in Christian radio broadcasts. My colleagues in Communication Arts tell me this is called "selective perception and retention," a process where one sees and remembers what that person is already thinking about and agrees with. However, it wasn't until I taught an 8:00 a.m. statistics class that I decided to take action in an organized and deliberate manner–devotions for that class would be instances where the mathematics we were studying reflected God. The success in that statistics class motivated me to continue the project with an 8:00 a.m. multivariable calculus class. In 2001, I decided to do weekly devotions for all of my classes with the same ground rules. I currently have devotionals connected to content in calculus, multivariable calculus, discrete structures, linear algebra, differential equations, and statistics. Collecting all of these snippets of mathematical reflections of God into a single Internet-based document is an ongoing process[2]; what follows are a few of these devotionals which I think broaden and enlighten understanding of theological concepts.

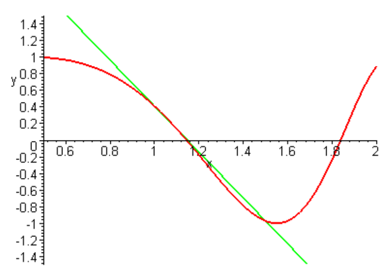
# 2 Christianity through a Mathematical Lens

## 2.1 Differential and Integral Calculus

**Secant Lines and Sanctification.** In differential calculus we study how a slope of a linear function can be generalized to the slope of a function whose graph is curved, creating the derivative of the original function. The definition of derivative uses a sequence of lines (secant lines) drawn through two points on a function that are approaching each other and a single point on the function curve. The derivative value or tangent line slope is defined to be the limiting slope value of this sequence of secant lines. See the figures below.

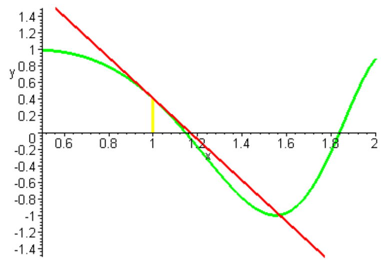



Secant line between 1 and 1.8




Secant line between 1 and 1.5

[2]See <www.trnty.edu/faculty/robbert/SRobbertWebFolder/ChristianityMath/index.html> for this document.



Tangent line to $f$ at $x = 1$

Once a person has been called to be a Christian, they are redeemed by Christ but not released from following the law of God. Christians are justified once but continue with the process of sanctification for the remainder of their lives. This sanctification process is like the limit process of the secant lines approaching the tangent line. There is one distinction between the concepts of sanctification and secant line limits, however. In the mathematical contexts, we accept results that are "sufficiently close," results that are in an $\epsilon$-neighborhood of the desired quantity. While in the quest for perfection, the "better" one becomes, the further they realize they are from satisfying all aspects of the law.

**Blessed Exponentially.** Elementary functions play an important role in calculus. The rate at which those elementary functions grow for increasing input values is one characteristic we study.[3] The fastest growing elementary function class is the exponential function class; functions in this class take variable powers of a fixed numerical base. The principle of exponential growth is exploited in savings plans (save early and often!) and modeled in growth of bacteria.

Christ tells us in Matthew 5:43–47[4] that we are to love our neighbors AND our enemies. We also read in Genesis 12:2-3 that God blessed Abraham so that "all peoples on earth will be blessed through [him]." Together the concepts of exponential growth and "blessed to be a blessing" tell us to "pay it forward," so that God's love for humankind and goodness can grow exponentially.

**Deceptions.** In calculus we use technology freely; in particular to produce graphical images with graphing calculators and computer algebra systems. Technology is not perfect, however, and those who use technology must be aware of times when the graphical images we see are not representative of the true nature of the object. We use mathematical experience and developed intuition to judge whether an image is flawed or deceptive.

Satan is the angel of light and his disciples masquerade as "servants of righteousness."[5] But we read in Matthew 24:24 that it is impossible for false Christs to deceive the elect.

[3] Applications of this growth analysis appear in algorithm complexity analysis in computer science. Exponential growth is "bad" in this instance.

[4] All scripture passages are taken from the New International Version of the Holy Bible.

[5] 2 Corinthians 11:13–15

We must follow the example of Jesus and use scripture as a standard against which to measure truth. We must also put on the full armor of God to protect ourselves from Satan's attacks.[6] In both situations, knowledge helps prevent deception.

**God's Zero Tolerance for Error.** Analytically finding the area between a curve and the horizontal axis is a primary topic in integral calculus. We learn that some curves are resistant to exact methods of area computation, so geometric estimation techniques are required. In every estimation problem, it is insufficient to find an estimate without also knowing theoretically how close the estimate is to the quantity we wish to estimate—this is finding an upper bound on the error. We deal with relations that look something like this,

$$|\text{Desired Quantity} \ - \ \text{Estimate}| \leq \text{Error bound.}$$

In most applied situations we can allow for a small error; if we're off by 0.00001, that might be okay.

There is an equivalent error analysis in comparison between our attempts to meet God's law and the perfection demanded by God's holiness. Here, God requires zero tolerance for error in order to be accepted into His kingdom. So the relation looks like this,

$$|\text{Standard of God's Law} \ - \ \text{Our imperfect actions}| \leq 0.$$

We are unable to meet this zero error bound, so on our own we cannot be accepted into the kingdom. However, Christ exchanged places with us—He put his perfect self in our place in comparison to God's Law and took our punishment of death. This makes us able to satisfy the zero tolerance for error.

## 2.2 Multivariable Calculus

**Approximations and Intentions.** Sometimes, analysis of a given relationship in multivariable settings is complex–too complex to be worth the effort of exact calculation. Instead, it is sufficient to use an approximation for the relationship that matches some but not all of the essential characteristics of the original relationship. This is modeled below by the planar approximation for the lumpy surface. To calculate values of the lumpy surface, we instead find a simple function (here a plane) that in a small window around the point the values will agree within some tolerance for error.

[6] Ephesians 6:10–17

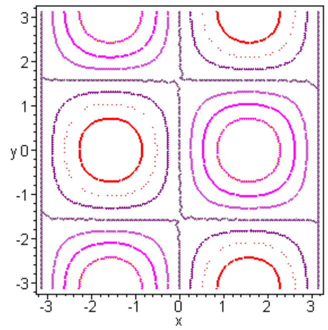
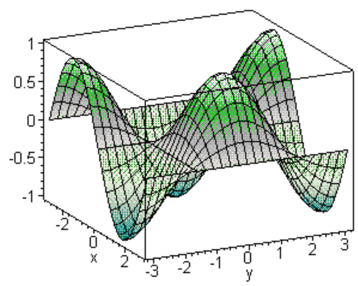
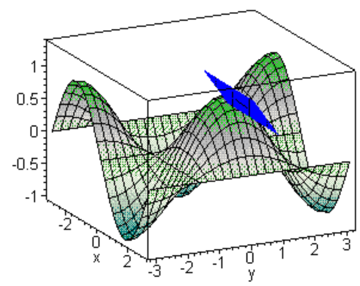



Figure 2: Level Sets for Lumpy Surface




A Lumpy Surface




Planar approximation to Lumps

This idea is somewhat like the relationship between the original beauty of God's creation and the world we see today. We are unable to see clearly the goodness–we see only in a "mirror dimly"[7] the approximation to the perfect creation intended by God. This gives us hope for what the new creation will be like when Christ returns again in glory!

**Stratification and Level Sets.** In the three-dimensional images you see above, the location of the hills and valleys are easy to see. Not quite as easy to see in the diagram are the saddle points that lay diagonally between two valleys and two hills. To better understand the characteristics of a function, students use level sets, a collection of two-dimensional graphs which gives detail about slices of the function using regularly spaced heights. Figure 2 contains a portion of the level sets for the lumpy surface above. The hills and the valleys in the level set plot are at the centers of the concentric "circles" while saddle points occur at the intersection of the boundary lines.

This idea of grouping the function input values by output values is a little like the stratification we see daily in human culture. There are those who are perceived to be at

[7]I Corin. 13:12

the highest level (e.g., Michael Jordan, Queen Elizabeth II) and those who we see at the bottom of the deepest valleys (e.g., death-row inmates, Al Qaeda terrorists). However, we learn in Colossians 3:10–11 that in Christ there is no such ranking. "Here there is no Greek or Jew, ..." and all Christians must "put on the new self, which is being renewed in knowledge in the image of its Creator." However, Christians are not clones of each other. Each is given a different configuration of spiritual gifts.[8] In this type of stratification, we rejoice in the distinctive service we can provide for God's kingdom.

## 2.3 Discrete Structures

**Conjecture and Proof: God's Will.** One thing that developing mathematicians must learn to do is to design new mathematical systems. At the Sophomore level, we begin that process within a system where students have a lot of experience—number relationships. Students look at simple patterns and try to generalize the relationships they see. The generalization created is called a conjecture. Conjectures are excellent first steps in the design of new mathematical systems; however, to be useful, the person must try to write a convincing argument demonstrating why the statement is true or find a counter-example demonstrating why the statement is false.

Developing Christians are taught to seek the will of God in making life-decisions. To be able to determine the direction God wishes us to go, we must form conjectures and reason to conviction of truth. Paul says in Romans 12:2, "be transformed by renewing of your mind. Then you will be able to test [conjecture] and approve [reason to conviction of truth] what God's will is ..."

$\exists!$ **God.** In the process of learning acceptable mathematical procedures for writing an argument which is convincing to other readers, we study predicate logic. Determining what portion of the entire collection will satisfy a relationship is one component of the argument. Mathematicians indicate the special cases of *all*, *at least one*, and *exactly one* with quantifier notation. If an open statement $P(x)$ is true for all valid replacements $x$, we write $\forall x, P(x)$. If the open statement $P(x)$ is true for at least one replacement, we write $\exists x, P(x)$. And, if an open statement is true for one and only one replacement, we write $\exists! x, P(x)$. Unique existence of a valid replacement is one of the most special cases to consider. A proof of this type of statement always requires two parts: first, you must show that *at least one* solution exists (i.e., existence of solution); then you must show that *not more than one* solution exists (i.e., uniqueness of solution).

Abraham and his descendants were chosen to be the first people on earth to be led to comprehend both aspects of the unique existence of God. One instance of the existence portion of God is found in the story of Moses meeting God in the burning bush. Here, God

[8] Romans 12: 6–8

reveals his name to Moses as evidence that He truly exists. God says, "I am who I am."[9]

Later, at Mount Sinai, Moses is given laws to train the infant-nation of Israel in the ways of God. In Deuteronomy 6:4, Moses recounts the uniqueness condition told him by God: "Hear, O Israel: The LORD our God, the LORD is one." Even though both aspects of the "$\exists!$ God" were provided to Israel from the time of the exodus, we know from Old Testament stories that the lesson was a difficult one for these chosen people to learn. I think even today we struggle with acknowledging God's unique existence, though few Christians will deny the truth of the statement.

## 2.4 Linear Algebra

**The Mark of a Determinant.** Systems of equations play an extremely important role in applied mathematics. A system of equations is a set of equations that are solved in tandem; solutions to the system must satisfy every equation individually. In these systems, complex relationships can be modeled. Relationships such as those between sectors of the United States economy, components of computer-aided design, and even the flight controls of the space shuttle can be modeled and examined in simulation—all without putting drivers of cars and astronauts at risk.[10] One method that mathematicians can use to determine whether a systems has a solution or not is by calculation of a matrix determinant number. If the determinant value is not zero, then a unique solution to the system exists. Though the determinant has theoretical value, its practical value is limited. It often takes more computational effort to find the value of the determinant than it does to apply common system solution algorithms.

This type of indicator exists in scripture as well–there are spiritual marks that indicate the bearer's allegiance to God or to Satan. Most people are familiar with the "mark of the beast" John describes in his vision in the book of Revelations. Here the mark of the beast was the number 666. However, there are several instances where God marks the faithful with a mark or a seal. In a vision, Ezekiel hears God instruct his assistant, "the man clothed in linen," to put a mark on the foreheads of those who have stayed allied with God. These persons were to be spared execution when God's vengeance was delivered.[11] Other instances of marks or seals of God are found in Galatians 6:17 and in Revelations 7:3. Except for the marks of Christ Paul describes in the Galatians passage, these marks are not practical; they only occur in a spiritual setting. This makes the connection to the determinant more striking.

**Generalization and Fulfillment.** Many concepts in mathematics are studied in a spiral. We study a concept in a conceptual setting, then generalize to a more complex setting

[9] Exodus 3:13–14. I especially enjoy the transcendence of God to time given within the Hebrew for this phrase–the phrase can be interpreted with past, present, and future verb tenses!

[10] See the chapter introductions of D. Lay's Linear Algebra book for more interesting applications!

[11] See Ezekiel 9:1–7

by increasing dimension, and then generalize again to an even more complex setting by selecting characteristics that appear useful and then removing all other aspects of the conceptual setting. One of the first times the second type of generalization occurs in the undergraduate mathematics curriculum is in the study of general vector spaces.[12] Students find the lack of a conceptual setting troubling; in fact, instructors of linear algebra often refer to students "hitting the wall" when they first encounter general vector spaces.

Christ also had trouble with his students "hitting the wall" during his ministry. One important aspect of Jesus incarnation was to teach people the meaning behind the law of Moses. His death and resurrection are described as the fulfillment of the Old Testament law. This generalization of the law is beautifully described by Jesus during the sermon on the mount.[13] A common phrase Jesus used during this sermon is "You have heard that it was said. . . ." Each of these phrases is followed by a generalization of an Old Testament law to include the intent behind the law: murder is generalized to include hatred, adultery is generalized to incorporate lust, and love for neighbors is generalized to love for all. The students of Jesus who "hit the wall" were the ones who thought they understood the law the best–the leaders of the Jewish faith.

## 2.5 Probability and Statistics

**Probability Defied.** The probability of success is determined mathematically by looking at the empirical ratio $\frac{\text{number of successes}}{\text{number of possibilities}}$, where the number of successes and possibilities are taken from a theoretical sample space. For example, if you want to calculate the probability that you will roll a seven with a pair of distinct dice, you note that there are 6 ways to roll a seven (1-6, 2-5, 3-4, 4-3, 5-2, and 6-1) from among all the 36 possible combinations. So the probability of rolling a seven is $\frac{6}{36} = \frac{1}{6}$. This ratio is always a number between 0 and 1, inclusive. The larger the number in this interval, the more likely the event is to occur. Less likely events have a smaller number in the interval.

God is able to beat the odds, however. He proved to Gideon that this is the case by systematically eliminating a large portion of his fighting men, selecting only 300 out of 32,000 to fight the Midianites.[14] Jesus also used probabilities to teach his disciples. He says in Matthew 19:24 that "it is easier for a camel to go through the eye of a needle[15] than for a rich man to enter the kingdom of God." So, the probability of salvation for a rich man is given as very small. He comforts his disciples in verse 26 of the same chapter by letting them know that God is more powerful than probabilities—"with God all things are possible."

[12] Instances of this type of generalization also occur in non-Euclidean geometry, in group theory, . . . .
[13] Matthew 5–7
[14] Judges 7:1-8
[15] Some Biblical scholars take the eye of the needle to be a small but busy gate into the city of Jerusalem.

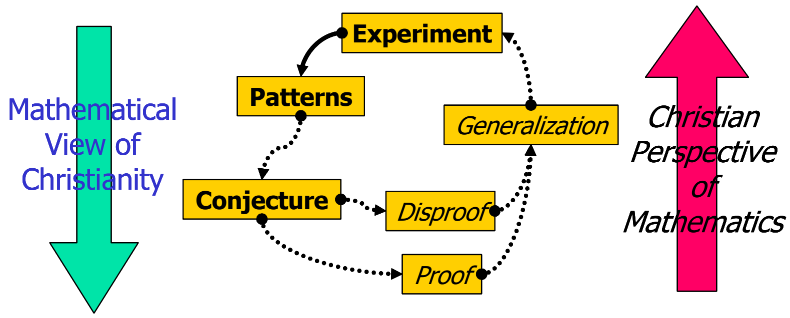



Figure 3: Classification of work in this paper

**Better than Average.** When we study numerical data in statistics, we organize the data, draw visual summaries, rank the data, and calculate measures of central tendency, among other things. With measures of central tendency, we calculate the mean (arithmetic average) and the median (middle value) to try to get a meaningful way to describe the data. Most teachers use mean and median and other descriptive statistics on test results to decide if students are performing as we expect in a class. Most students are happy if they are near or above the middle of the pack on a test score.

However, God want more than a middling performance in terms of our Christian behavior. He wants our behavior to be far away from that of the average "good" person. In Revelations 3:14–22, John is told to write to the Laodicean church that their lukewarm performance is causing them to be in danger of eternal rejection. Likewise, we must not be content with our current state as a Christian. We must always work to be even more "extreme" for God.

## 3 Conclusion.

In conclusion, mathematics and Christian thinking are not separate entities. Each one informs the interpretation of the other, but identifying ways in which each informs understanding of the other is not of equal difficulty. The work described in this document fits into one small part of the solution, the experimentation stage of a mathematical view of Christianity, i.e., the direct problem. I am hopeful that these experiments will lead to pattern identification and the establishment of plausible conjectures (see figure 3 ).

There are many questions that are natural outcomes of this structural model. For example,

- Are there types of theological constructs that recur frequently?
- Are there theological constructs that elude mathematical insights?

- Does either the direct or inverse problem direction allow for existence and/or uniqueness discussions?
- Should additional aspects of mathematical investigation be added to the model? Aspects one could easily argue to include in the model are application or interconnectedness.
- Does the philosophy of mathematics and the cultural and historical aspects of creation fit in this model? If so, where?

Obviously there is much more work to accomplish in both directions of this inverse problem. But that is one of our main purposes as children of God who love mathematics—we must continually look for ways to become more closely identified with God's revealed truth.

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