

History of Mathematics in the Service of School Mathematics Education

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OUTLINE

Textbook Project Description
HoM for Math Educators
Greek Ratio & Proportion
Numerical Theory of R&P
Educational Ramifications
Geometric Theory of R&P
Use of Geometric R&P
More Educational Ramifications
Concluding Remarks

TEXTBOOK PROJECT DESCRIPTION IN BRIEF

Course for Prospective Middle School Mathematics Teachers
Curricular Identity Crisis: Changing Educational Scene
Refocused, Using History of Mathematics
Need for Class Materials
Homemade Exercises
Extensive Outlines
Textbook Sections: in process

HISTORY OF MATHEMATICS FOR MATHEMATICS EDUCATORS

Cultural Benefits
Cultural Matrix of Mathematics
Multi-Cultural Contributions
Embedding Mathematics in an Integrated Social Studies Unit
Views of the Nature of Mathematics
Origin and Development of Ideas and Techniques
Ontogeny Recapitulates Phylogeny?
Cognitive Development, with Cultural Differences
HoM Alerts Educators to Difficulties and Distinctions
HoM Alerts Educators to Developmental Prerequisites
Pedagogical Implications

GREEK RATIO AND PROPORTION

Meaning of Terms
Origins and Early Development
Pre-Greek Use of Ratio and Proportion
Slope of inclines
Work rates
Strength of bread/beer

Greek Theory of Ratio and Proportion
Cultural Context: Rational Explanation/Foundation
Numerical Theory of Pythagoreans [Euclid VII]
Geometric Theory of Eudoxus [Euclid V]

Numerical Theory of Ratio and Proportion

Ratio

Relation in respect of size between two quantities [V, Def.3]
Not a fraction; and certainly not a number

Proportion

Sameness of Ratios? (Euclid V)
Proportional when same part, parts, or multiple [VII, Def.20]

Proportion Explicated

Part: when it measures the whole; submultiple [VII. Def.3]
Parts: smaller, but not a part [VII, Def.4]
Same Part: same multiplicative relation, same unit fraction
Ex: 3 is the same part of 12 that 5 is of 20: the fourth-part
Same Parts: same number of parts of same part-type
Ex: 8 is the same parts of 12 that 10 is of 15: 2 third-parts

Proportional numbers

Ex: $3:12 :: 5:20$; $8:12 :: 10:15$

Modernized Criterion: proportional iff same fully reduced form

Educational Ramifications of Numerical Theory of R & P

Ratio

Meaning of ratio as a multiplicative relationship is central
Connection between ratio and fractions/numbers: takes time?
Comparison to MSM Ed textbooks: ordered pair definition
Far too abstract for learning what a ratio is
Wrong; not abstract enough for a formal treatment

Proportion

Meaning of proportion as same fully reduced ratio is intuitive
Euclid's criterion stays within realm of ratios/fractions
Comparison to MSM Ed textbooks: cross multiplication criterion [VII.19]
Less fundamental?
But easy to apply

Geometric Theory of Ratio and Proportion

Ratio

Relation in respect of size between two quantities [V, Def.3]
Magnitudes have a ratio iff they are capable, when multiplied, of exceeding one another [V, Def.4]
Homogeneity requirement
Not infinitesimal/infinite with respect to one another
Can be incommensurable with one another (key!)

Proportion

Proportional: sameness of ratios [V, Def.6]

Same Ratio: any common multiple of the first and third alike exceed, equal, or fall short of any common multiple of the second and fourth respectively [V, Def.5]

Heterogeneity permitted in proportions

Use of Geometric Ratio and Proportion

Mathematics

Area

Volume

Similar figures; indirect measurement

Kinematics

Uniform motion/constant speed: $d_1:d_2 :: t_1:t_2$

Mechanics

Law of Lever: $d_1:d_2 :: w_2:w_1$

Centroids of regions

Beginnings of quantitative science/mathematization

Educational Ramifications of Geometric Theory of R & P

Ratio

Homogeneous ratios: fit multiplicative meaning of ratio best arise first? simplest to understand?

Heterogeneous ratios: rates arise late (early modern era) easier for today's students? still conceptually complex?

Comparison to MSM Ed textbooks: little attention paid to the difference between homogeneous and heterogeneous ratios

Proportion

Heterogeneous magnitudes can be related by abstract magnitude-neutral ratios between homogeneous quantities

Cross multiplication criterion leads to compound magnitudes

Ratio and proportion: the backbone of mathematical science

CONCLUDING REMARKS ON PROJECT

Value of History of Mathematics for (Middle School) Mathematics Teachers

Overview and Perspective

Background and Breadth/Enrichment

Comprehension of Topics' Meaning and Significance

Value of History of Mathematics for the (Middle School) Classroom

Overview and Perspective

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Questions or Suggestions?