The Topology of Harry Potter:
Exploring Higher Dimensions in Young Adult Fantasy Literature †

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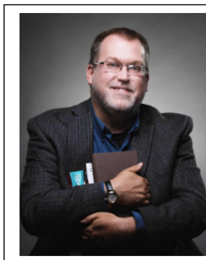
Having completed her Master's Degree in mathematics at the University of Wisconsin-Milwaukee, Alexa Schut is now an Assistant Professor of Mathematics at Culver-Stockton College in Canton, Missouri. With Bill and Dave, she coauthored a related article on Harry Potter and higher dimensions in the *Journal for Adolescent and Adult Literacy*. Outside of the classroom, she enjoys spending time hiking with her husband. She is a huge fan of Harry Potter and is a Slytherin.

Dave Klanderman has been a Professor of Mathematics at Trinity Christian College and begins as a Professor of Mathematics Education at Calvin College in the Fall of 2018. His research interests range from learning trajectories for length, area, and volume measurement to inquiry-based learning. He recently coauthored *A Pleasure to Measure*, a book of classroom activities for elementary school teachers. Although he has only read some of the Harry Potter books, his daughter assures him that he is a Ravenclaw.



Bill Boerman-Cornell is a Professor of Education at Trinity Christian College who researches how literacy differs within the academic disciplines. He is one of the authors of Graphic Novels in the High School and Middle School Classroom: A Disciplinary Literacies Approach, published by Rowman and Littlefield. His research has appeared in a wide range of journals, including *Educational Leadership, The History Teacher*, and *Bookbird*. While he empathizes with all the houses, Bill lives in Gryffindor.

Abstract

As one of the most beloved series in children's literature today, the Harry Potter books excite students of all ages with the adventures of living in a magical world. Magical objects (e.g., bottomless handbags, the Knight Bus, time turners, and moving portraits) can inspire generalizations to

† In this paper, instead of providing a rigorous definition for *dimension*, we refer to a more familiar and less technical use of the term. For example, polygons are normally thought of as two-dimensional, while polyhedra are three-dimensional, etc. Although precision in mathematical terms is necessary, we believe that students will be more motivated to learn these rigorous definitions after exploring less technical (and arguably more interesting) analogies.

mathematical concepts that would be relevant in an undergraduate geometry or topology course. Intuitive explanations for some of the magical objects connect to abstract mathematical ideas. We offer a typology with a total of five categories, including Three Dimensions in Two Dimensions, Higher Dimensions in Three Dimensions, Two and Three Dimensional Movement, Higher Dimensional Movement, and Higher Dimensional Traces. These categories attempt to explain supernatural events from the wizarding world using mathematical reasoning in order to increase engagement in topics from topology to differential geometry. Our pedagogical approach is to pique student interest by linking these abstract concepts to familiar examples from the world of Harry Potter. Put on your Ravenclaw robe or Gryffindor scarf and join us!

1 Introduction and Overview of Research Project

The Harry Potter books by J.K. Rowling are not only incredibly popular, but they also provide wonderful analogies for higher dimensional thinking. Many of the experiences of a typical witch or wizard include magical objects, such as handbags enlarged via engorgement charms, time turners (which come in handy for taking two or three classes that meet at the same time), or moving portraits whose subjects can interact with people outside the frameall of these can in fact inspire generalizations to mathematical concepts that would be fitting in an undergraduate geometry or topology course. Many of the magical properties demonstrated in these books have a reasonable explanation if one considers the notion of higher dimensions or other abstract mathematical concepts. Our goal in this paper is to use analogies to motivate interest among students and to explain complicated and precise ideas in a more intuitive and less formal manner. That is, these vignettes from the novels serve as an anticipatory set, i.e. an introductory activity for the lesson that creates enthusiasm among students for later rigorous definitions, theorems, and further mathematical examples.

As part of a senior undergraduate research project, Alexa Schut explored the links between higher dimensions and young adult fantasy literature. Dave Klanderman and Bill Boerman-Cornell served as faculty mentors for Alexa and all three collaborated to find numerous examples of higher dimensional concepts in a wide variety of young adult fantasy literature. These include not only the Harry Potter novels of J.K. Rowling, but also the *Chronicles of Narnia* and the *Space Trilogy* by C.S. Lewis, *Flatland* by Edwin Abbott, and *A Wrinkle in Time* by Madeleine L'Engle. Based upon this research (which continued as Alexa pursued graduate degrees), five categories emerged that describe different types of dimensional concepts. Although these categories apply to all of the aforementioned novels, and although we included references to them in the typology chart below, for the sake of clarity, we will restrict our discussion in this paper to the Harry Potter series.

This first phase of the research led to an article published in the *Journal of Adolescent & Adult Literacy* that describes ways for middle school and high school mathematics and English teachers to entice strong mathematics students to explore higher dimensional concepts in the wizarding world of Harry Potter. It also provides the motivation for avid Harry Potter readers to desire a deeper understanding of these higher dimensional concepts linked to recommended topics in the middle school and high school mathematics curriculum.[1]

The next phase of research began with an observation by Sarah Klanderman, currently a Ph.D. student

[1]BoermanCornell, W., Klanderman, D., & Schut, A. (2017). Using Harry Potter to Bridge Higher Dimensionality in Mathematics and HighInterest Literature. Journal of Adolescent & Adult Literacy, 60(4), 425-432. Available at http://onlinelibrary.wiley.com/doi/10.1002/jaal.597/full

3D in 2D	Higher D in 3D – Pocket Dimensions	3D/2D movement	Higher D movement	Higher D Traces
~ Portraits (HP) ~ Newspaper (HP) ~ Chocolate Frog Cards (HP) ~ 2D world (WIT) ~ Books (EA) ~ The first conversation between Mr. Square and his grandson about how a square moving parallel to itself must create a geometric shape. (Flatland) ~ When Mr. Square puts squares on top of each other to create a solid while in the 3D world. (Flatland) ~ Tom's Journal (HP) ~ Room of Requirements (HP) ~ The Veil (HP -5th) ~ Doors in the Department of Mysteries entrance (HP) ~ Pensieve (HP) ~ Portrait in Hogs Head (HP) ~ Scar (HP)	~ Hermione's Bag (HP) ~ Tent (HP) ~ Train Station in the pillar (HP) ~ Wands (HP) ~ Eldila (Space T) ~ Oyarsa (Space T) ~ Diagon Alley (HP) ~ Sword in the Sorting hat (HP) ~ The fourth dimension as seen only by the inside of a 3D being as Mr. Square proposed to the Sphere. (Flatland) ~ Carriage (HP-4th) ~ Moody's Trunk (HP) ~ Room of Requirements (HP) ~ The Veil (HP-5th) ~ Doors in the Department of Mysteries entrance (HP) ~ Pensieve (HP) ~ Horcruxes (HP) ~ Scar (HP) ~ Weasley's Car (HP) ~ Ministry of Magic Underground (HP) ~ Vanished Objects (HP-7th) ~Woods between Worlds (Narnia)	~ Grim Place (HP) ~ Marauder's Map (HP) ~ Knight Bus (HP) ~ Car (EA) ~ Mirror of Erised/Sorcerer's Stone (HP) ~ Portrait in Hog's Head (HP) ~ St. Mungo's Mirror Entrance (HP)	~ Tessering (WIT) ~ Time Travel (WYRM) ~ Times linearity or lack of (WYRM) ~ Chromogaurd (EA) ~ The idea that if a four dimensional being is moving then we would see at as progression in time from a 3D view. (Flatland) ~ Train (HP) ~ Dobby's popping (HP) ~ Floo Powder (HP) ~ Time Turner (HP) ~ Portkey (HP) ~ Ship (HP-4th) ~ Triwizard trophy (HP) ~ Vanishing Cabinet (HP) ~ Gamp's 5 laws of magic (HP) ~ Vanishing Objects (HP-7th) ~ Apparating(HP) ~ Rings (Narnia) ~Food from a drop of liquid (Narnia)	~ Magic (HP) ~ Ghost (HP) ~ Resurrection Stone (HP) ~ Eldila (Space T) ~ Oyarsa (Space T) ~ Hypercube (video) ~ Leaky Cauldron (HP) ~ Stare of the Basilisk (HP) ~ Thestrals (HP) ~ Money in Gringotts that duplicates (HP) ~ Limbo state (HP) ~ The fact that when the Sphere enters Mr. Square's house all Mr. Square sees is the movement of a circle not the whole sphere. (Flatland). ~ "Ghosts" that appeared when Harry and Voldemort's wands meet (HP-4th)

Figure 1: *Typology Chart of Categories of Links between Young Adult Fantasy Literature and Higher Dimensions. Note:* ***HP*** *= J.K. Rowling's Harry Potter series (book number where noted),* ***WIT*** *= Madeleine L'Engle's A Wrinkle in Time,* ***Narnia*** *= C.S. Lewis's Chronicles of Narnia series,* ***SPACE T*** *= C.S. Lewis's Space Trilogy,* ***WYRM*** *= Rebecca Stead's When You Reach Me,* ***EA*** *= Jasper Fforde's Eyre Affair,* ***Flatland*** *= Edwin A. Abbott's Flatland,* ***Video*** *= Hypercube YouTube video*

in algebraic topology at Michigan State University. She noted that many of these five categories of dimensional concepts would lend themselves well to in-class examples in an undergraduate topology course. Sarah joined our research team at this juncture, and we plan to discuss three of these topological applications in detail in this paper.

The following sections of the paper will introduce each category, provide a comprehensive description for one specific example within the category along with a list of other examples, and, where appropriate, a discussion of mathematical concepts from topology that provide natural links from the Harry Potter novels to the undergraduate mathematics classroom. Here, the categories attempt to explain supernatural happenings in the wizarding world using mathematical reasoning in order to motivate topics in courses in topology or differential geometry. The descriptions in this article are designed to be only an introduction of the mathematics in order to clarify the literary connection to Harry Potter but can easily be supplemented with textbooks such as those referenced in this article. The goal is to pique students' interest by introducing upper- or graduate-level topics within the context of popular novels in order to motivate student understanding.

2 Category 1: 3 Dimension in 2 Dimensions

The first category of *Three Dimensions in Two Dimensions* can be briefly explained as three-dimensional objects that are in fact bounded in two-dimensional space. In *Harry Potter and the Sorcerer's Stone*, we see examples of this category as manifested in newspaper photos, wanted signs, Tom Riddle's journal, and the portraits of Hogwarts: Harry's legs were like lead again, but only because he was so tired and full of food. He was too sleepy even to be surprised that the people in the portraits along the corridors whispered and pointed as they passed.[2] These moving portraits, not unlike our own flat screen televisions, could be used to introduce topics such as stereographic projections, the real projective plane, or diffeomorphisms as defined using charts and atlases. Although the painting or photo is inherently contained in a two- (lower-) dimensional canvas like the projection or local neighborhood, the depicted scene (original space) is inherently three- (higher-) dimensional, which nevertheless allows them to move even beyond the bounds of their own painting into neighboring ones.

3 Category 2: Higher Dimensions in 3 Dimensions/Pocket Dimensions

The second category is *Higher Dimensions within Three Dimensions*, often more easily understood as pocket dimensions. Similar to the first category, this topic covers higher dimensional objects that move, operate, and exist within the confines of three dimensions. A well-suited example from the Harry Potter books is Hermione's bag, which particularly comes in handy during their travels in the seventh book, *Harry Potter and the Deathly Hallows*.[3] Because the handbag has an engorgement charm on it, it has the capacity to store many times more than it naturally should. In this way, the bag is a pocket dimension of sorts, containing more in its interior than is suggested by its small outer lining.

One way that we may consider this idea of pocket dimensions is as an outer reality (i.e. X = the outer appearance of Hermione's bag), which is homotopy equivalent[4] to that which is contained in the pocket dimension (i.e. Y = the inner contents of Hermione's bag). According to Hatcher's *Algebraic Topology*, "a map $f : X \to Y$ is called a **homotopy equivalence** if there is a map $g : Y \to X$ such that $f \circ g \simeq 1$ and $g \circ f \simeq 1$. The spaces X and Y are said to be homotopy equivalent or to have the same homotopy type."[5] So although it isn't clear how one would define the continuous map from the inside of Hermione's bag to the outer beaded appearance in this situation (hence, why the idea is to intrigue students so that they are encouraged to bring their own imagination and investigate more), it's still clear that we have two spaces between which we cannot seem to visually understand their equivalence.

Other examples from the books of this homotopy equivalence of a space and what could be considered its pocket dimension would be Platform $9\frac{3}{4}$ (the train station seems to be contained within a single support pillar in King's Cross) and the small tents that Harry and Hermione stay in along with the majority of the Weasley family during the Quidditch World Cup.[6] Each tent has been charmed to be bigger on the inside than the outside perimeter would suggest,[7] able to hold a small kitchen, bunks, and, in one case,

[2] J. K. Rowling (1997) Harry Potter and the Sorcerer's Stone New York: Scholastic Inc. p. 128

[3] J. K. Rowling (2007) Harry Potter and the Deathly Hallows New York: Scholastic Inc.

[4] We recognize that a variety of notions of equivalence could instead be introduced here, but opt for homotopy equivalence because it doesn't require us to define metrics in a fictional universe, and it is also a topologically specific concept that often seems obscure to students upon first encounter.

[5] Allen Hatcher (2001) Algebraic Topology https://www.math.cornell.edu/ hatcher/AT/AT.pdf

[6] J. K. Rowling (2000) Harry Potter and the Goblet of Fire New York: Scholastic Inc.

[7] Perhaps using technology similar to the dimensional transcendentalism of the TARDIS (from the BBC television show

eight inhabitants (Harry, Ron, Bill, Charlie, Percy, Fred, George, and their father). Similarly to how we think of Hermione's purse, we may view one of these four-person tents to be a space X while their vast interiors form a space Y. However, because they occupy the same space via a pocket dimension, the two spaces are in this sense homotopy equivalent.[8]

4 Category 3: 3D/2D Movement

The third category includes situations in which a three-dimensional object must transform into a two-dimensional object, or vice versa. The Knight Bus provides transportation for wizards who are looking for a different pace than apparition or Floo powder. This vehicle is able to cover large distances quickly, even in heavy traffic conditions. The secret lies in the ability of the Knight Bus to flatten itself to squeeze between large vehicles traveling on the same road in both directions. In this sense, it temporarily transforms into a two-dimensional object before popping back out into its more typical three-dimensional form. It might be helpful to think of origami constructions that can be three-dimensional after multiple folds. However, the same object can be flattened back into a two-dimensional object by reversing these folds.

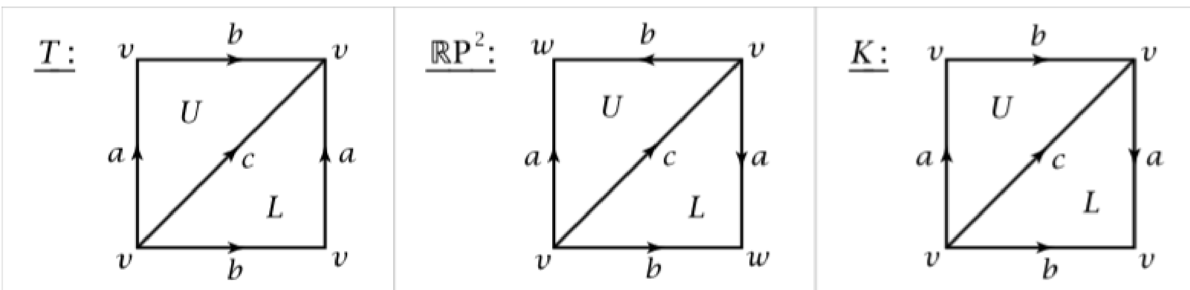


Figure 2: *Identification spaces of the torus, real projective place, and Klein bottle*[9]

From a more mathematical standpoint, this category links to the topological concept of an identification space. Identification spaces allow mathematicians to view higher dimensional objects through the analysis of two-dimensional figures that encode the higher dimensional information via gluings. For instance, a torus can be constructed from its identification space by gluing the sides labeled a to form a cylinder, and then gluing b to form its familiar donut shape. Unlike the torus or the other orientable surfaces shown in Figure 2, the real projective plane and Klein bottle live in four dimensions[10]; therefore we must use identification spaces in order to visualize them to the best of our abilities.

Other examples of this category would be the Marauder's Map, which depicts the movements of the

"Doctor Who"), which is accomplished via trans-dimensional engineering.

[8]As an example, the authors suggest implementing this motivating example for homotopy equivalence as follows. Near the beginning of an undergraduate topology course, an opening question could be, "When do we have two spaces that are equivalent?" Students may have a variety of answers, including ideas ranging from trivial labeling equivalences (e.g. "my bedroom" vs. "the room in which I sleep") to homeomorphisms/diffeomorphisms, etc. However, for a nontrivial example of distinct but co-existing spaces, the instructor could show a clip of Hermione pulling a large painting from her small handbag (thereby indicating its larger interior). The instructor could then move toward a more formal definition of homotopy equivalence, perhaps by using more standard mathematical examples (e.g. Möbius band and circle).

[9]Allen Hatcher (2001) Algebraic Topology https://www.math.cornell.edu/hatcher/AT/AT.pdf

[10]Technically, the fourth dimension is merely the smallest Euclidean dimension into which the Klein bottle embeds. However, due to our loose usage of the term *dimension*, one would first expect to see the Klein bottle "living in" four dimensions.

inhabitants of Hogwarts on a two-dimensional map, and 12 Grimmauld Place, the location of the headquarters of the Order of the Phoenix, which is a large apartment flat that exists in the space between two apartments which adjoin each other. This 3D/2D movement could also be extended to include the mirror of Erised and the appearance of the Sorcerer's Stone and the St. Mungo's Mirror Entrance.

5 Category 4: Higher Dimensional Movement

The fourth category also describes movement across different dimensions. However, these movements require access to a fourth or another higher dimension. Higher dimensional movement involves the idea of stretching and distorting to travel while still remaining equivalent to your original form, whereas higher dimensional traveling is the idea of entering higher dimensions in order to travel at greater speeds. Both of these ideas can be found throughout the Harry Potter books, including traveling via the Floo Powder network, taking portkeys or apparating, Dobby's popping/house elf apparition, vanishing objects, the train that carries wizards to Hogwarts, and Hermione's time turner.

As with any over-achieving student, Hermione decides to take all available classes in her third year, despite their overlapping schedules, and is able to do so with the aid of a time turner, which allows her to travel back in time in order to attend three courses during the same time period. Although the notion of time travel is common throughout science fiction, we emphasize it here in connection to higher dimensional movement in order to introduce the concept of invertibility. In order to rationalize the necessity of a higher dimension, it may be helpful to imagine an ant at one end of a long rope. If restricted to two dimensions, it would be very time consuming for the ant to move from one end of the rope to the other. However, if the two ends of the rope are picked up into a higher (third in this case) dimension, then the ends can be brought next to each other, the ant can travel between the ends of the rope in essentially no time at all.[11]

From a more mathematical standpoint, this category links to the concept of invertible functions. According to Wolfram MathWorld, an object is invertible if it admits an inverse. [12] In the case of time turners, the inverse requires the traveler to live back through time to the original time of departure. Via this pseudo-mathematical explanation, the invertible map has a specified return trip due to a time reversal charm that can have serious consequences if the travelers attempt to somehow interact with themselves or change their own experience of the past (as seen in the shenanigans that occur in *Harry Potter and the Cursed Child*). These bounds on how the traveler can relive the past are similar to how, for practical reasons, we require a function to be bijective in order to define its inverse. In this sense, such an inverse is limited and, extending back to Harry Potter, the invertibility of time travel is restricted to reliving the past in the Harry Potter canon (unlike in the *Back to the Future* films for instance).

6 Category 5: Higher Dimensional Traces

As in Abbott's beloved novel *Flatland*, the Harry Potter books explore the concept of higher dimensional traces through ghosts, the resurrection stone, duplicating money within Gringotts vaults, and the "shadows" that appear from Harry and Voldemort's wands when they meet in *Harry Potter and the*

[11] As described in *A Wrinkle in Time*

[12] Margherita Barile From MathWorld A Wolfram Web Resource, created by Eric W. Weisstein http://mathworld.wolfram.com/Invertible.html

Goblet of Fire.[13] Similarly, in *Harry Potter and the Deathly Hallows*, as Harry confronts Voldemort for a final time, he uses the resurrection stone that Dumbledore bestowed upon him and surrounds himself with the shades of his parents, Sirius, and Lupin: "They were neither ghost nor truly flesh, he could see that. They resembled most closely the Riddle that had escaped from the diary so long ago, and he had been memory made nearly solid. Less substantial than living bodies, but much more than ghosts, they moved toward him, and on each face, there was the same loving smile."[14] These traces that seem ethereal in our dimension are perhaps but shadows of beings that exist on another plane, as if we are only seeing a reduced version based on what is available in three dimensions.

7 Conclusion and Next Phase of Research

As the above discussion has documented, there are numerous examples of higher dimensional concepts in the wizarding world of Harry Potter. Some of these examples would serve as motivating examples for concepts in an undergraduate topology course. Although there are limits to these links, a topology professor would be well served to include such examples during an introduction of these more abstract mathematical ideas. As we mentioned earlier, our analysis of young adult fantasy literature extends beyond the Harry Potter novels. Our next phase of research will be a more thorough analysis of the works of C.S. Lewis, Madeleine L'Engle, and Edwin Abbott, among others. This research will build on the prior work of a fellow ACMS member.[15]

[13] J. K. Rowling (2000) Harry Potter and the Goblet of Fire New York: Scholastic Inc.
[14] J. K. Rowling (2007) Harry Potter and the Deathly Hallows New York: Scholastic Inc.
[15] Neuhouser, David. (1995). Higher Dimensions in the Writings of C.S. Lewis. Faculty Dialogue, 161-176.