Mathematics as Poesis: A preliminary project report

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There are two basic viewpoints in the philosophy of mathematics. The first is the classical viewpoint that mathematics primarily entails discovery. Essentially, mathematical truth exists independent of the activity or thought of mathematicians. This existence may be *a priori* or empirical. The second view is that mathematics is primarily a creation of the human mind. There are many schools of thought within both of these categories, including Platonism, naturalism, logicism, intuitionism and formalism.

In Plato's philosophy the ideal world is the world of patterns or archetypes. The world of experienced phenomena is only an imperfect reflection of the ideal world. Platonism in the philosophy of mathematics does not make such strong claims about the ideal, but it would at least assert that mathematical objects have genuine existence independent of humans. Mathematical objects are neither ideas nor material objects, but exist on another level. This is one of the classical views of mathematics, which was the accepted view for centuries. This view has been severely challenged in the modern philosophy of mathematics. However, many, if not most, mathematicians still hold this view, at least in their practice of mathematics. They almost invariably talk of their mathematical results as discoveries. Paul Erdős, arguably one of the greatest mathematicians of the twentieth century, and certainly one of the most prolific mathematicians in history, often gave the following observation: "God has a book containing the most elegant proofs of all mathematical theorems. You don't have to believe in God, but you must believe in the book." Some mathematicians, such as Alain Connes [1], explicitly defend the Platonic view of mathematics. Many, if not most, Christian mathematicians have a very Platonic view of mathematics. For Christians, the ideal form is usually placed in the mind of God in some fashion, whether it is that all of the truths of mathematics are already determined in God's mind, or we are discovering how God's mind works, or some variant of these. Of course, this is a very simplistic representation of a number of very well thought out and developed points of view, as seen for example in [5].

Another major theme in the philosophy of mathematics is naturalism. A naturalist viewpoint is empirical, rather than *a priori*. This concept seems to be gaining favor among some mathematicians, since it seems to explain the applicability of mathematics. The most credible modern naturalist viewpoints claim, in essence, that human minds arose out of a process of natural selection, and hence naturally reason in ways that reflect the world around us. This seems to be the position of Changeux [1], who takes a biological approach to mathematical truth and reasoning. This approach to the philosophy of mathematics in many ways potentially bridges the gap between realism and constructivism. Since our minds are a product of the process of natural selection, the mathematics we construct naturally results in the discovery of physical laws of nature. This viewpoint sees mathematics as the creation of an independent human reason, and hence has no need for a concept of God. A counterargument to the natural selection thesis is given by Polkinghorne in [7] and Steiner in [8], in which it is argued that some of the greatest successes of mathematics are quite counter-intuitive.

The constructionist schools of thought typically have difficulty accounting for the applicability of mathematics to the real world, the so-called "unreasonable effectiveness of mathematics." The naturalist school deals quite well with this problem up to the point of how well mathematics models natural phenomena of which we have no experience (as discussed by Steiner [8]), but does not have room for God. This violates one of our basic premises, that "every discipline must be framed by a theological perspective; otherwise these disciplines will define a zone apart from God, grounded literally in nothing." [6, p.3].

The Platonic viewpoint can be very satisfying, in the sense that it is comforting to believe that we are discovering eternal truths or the mind of God, but it leaves very little room for the creative and aesthetic sense of the very human mathematicians who do mathematics and the interaction of these mathematicians with a creative God. Hence, this viewpoint misses the genuine creativity that is taking place in mathematical research and application. We do not intend, however, to remove this from the foundational work of God, but to say that we are participating with God in the creation of mathematics.

We are interested in developing a view of mathematics beginning within a Christian perspective. This philosophy will incorporate individuals' creativity and the mathematical community's aesthetic sense, both of which we see as essential to being made in the image of God. In this view, mathematicians are not merely uncovering God's treasures, but are fully functioning as mathematically "procreative" agents. We use the term procreative to emphasize that mathematicians are genuinely contributing in the process of creation, but that there is no such thing as purely human reason. God is active in the realm of reason (Logos) as much as in revelation. Both are aspects of participation of the mind of God. Thus, "to reason truly one must be already illumined by God, while revelation itself is but a higher measure of such illuminations" [6, p.24]. Reason, as a part of the created order, is essentially good, has been corrupted by the fall and continually depends upon God's sustaining grace. This basic theological truth provides an essential foundation that is missing within Constructivism and Naturalism as they are typically expressed. The secularization that these schools of thought have imposed leaves them "grounded literally in nothing." Even the Platonists have exposed themselves to this secularization, as evidenced by the previously stated quote from Erdős. This places us in direct opposition to many modernists, who have supposed that human reason is capable of independently and completely understanding the world. It is also in opposition to many post-modernists, who suppose that there is no grounding for our understanding, since they are merely human inventions that stand alone.

Our second basic theological premise is that our relationship with God is participatory. There is a genuine human contribution. This is due to the fact that there was a kenosis of God in the creation of the universe. In other words, God empowers his creation with life by limiting his control. God's self-limitation is not a lock of sovereignty; it is God's choice of how to exercise his sovereignty. For humanity, this includes intellectual and spiritual life, carrying the impression of God's image, with the ordinance to be fruitful, with the ability to gain knowledge, and the freedom and responsibility to make choices. Hence, we participate with God in the ongoing development of his creation and in the working out of his purposes in the world.

Hence, from these first two premises we see that neither God nor the human is absent in the development of mathematics.

Our third theological notion is that of poesis, or the art of making. God is a creative God and is continually creating. In developing the realm of mathematics, we are co-creating with God. Just as an artist, musician or author may be seen to be in partnership with God in the creation of beauty, so mathematicians may be seen as in partnership with God in creating the discipline of mathematics. This creative process takes many different forms. While there are certainly those truly great, obviously creative leaps in mathematics that create new realms, fields or disciplines within mathematics, the vast majority of mathematics being created is not of novel invention, but is a process of fleshing out the details. In fact, it may be understood that novel invention is important by virtue of the fact that that it gives others the space to do much useful work. Hence, no new structures or theories of mathematics are being defined, but the structures that have already been defined are in the process of being filled out. The distinction being made here might be called the difference between the actual and the potential. When a new structure or field is first defined, it is certainly created. But, once the axioms and definitions have been laid out some would say the structure has been created, the rest is just following the rules of logic and proving what the axioms and definitions imply. We would argue that the potential has been created but not actualized, and there is still a creative and aesthetic aspect to the actualization of mathematical structures. This process is parallel to a similar process in art, which begins with creative inspiration and then moves to the creative discipline to incarnate the idea.

Another implication of this partnership between God and humanity is that it simultaneously accounts for both the poesis of mathematics and the applicability of mathematics. In [8], Steiner argues that the applications of mathematics truly do have an "unrealistic effectiveness" and hence that the universe is "user friendly." We assert that this should be seen as a natural consequence of the fact that the creative foundation of mathematics is developed in partnership with the original creator of both the mathematicians and the entities being modeled. This may be seen as bridging the gap between the creation of mathematics and the discovery of patterns in nature. As Polkinghorne points out in [7], "the reason within and the reason without have a common origin in this deeper rationality which is the reason of the Creator." We thus leave no room for the secularization of mathematics. It is necessarily grounded in the Creator.

Given the assumption that every creative act is guided by aesthetic judgment we then may conclude that the heart of the mathematical endeavor is aesthetic. Harold Heie argues, in his essay "Mathematics: Freedom within Bounds" [4], that there are two primary functions of mathematical activity, instrumental and aesthetic. It could be argued that there is an underlying aesthetic component to the instrumental function as well, (at least Heie's second type of instrumental function); that is, applied mathematics also has an aesthetic foundation. This includes, for example, the aesthetic appeal of the way in which mathematics can be applied in novel and even surprising ways.

A key concept in explicating the connections between aesthetics and the applicability of mathematics lies in the concept of *mimesis*, or representation. In *Works and Worlds of Art* [9], Nicholas Wolterstorff emphasizes the fundamental role of representation in art. He argues that although representation is not essential, it is both pervasive and fundamental in art. Also, representation is not merely about symbols and their relationship to entities that they symbolize; rather, it fundamentally involves the human activity of "world projection."

> Representation is commonly understood to be a relation between symbols of a certain sort and that which they symbolize. I suggest, instead, that at its root representation is an *action* performed by human beings. ... Secondly, the heart of representation, no matter in which of the arts it occurs, lies not in composing a copy of the actual world but rather in using some artifact to project a world distinct from our actual world. Representation, or mimesis, is world projection. [9, Preface]

This approach to art cares little about the inherent experience of creation for the artist, and even less about the contemplative aesthetic experience for the observer. It also markedly contrasts with a generally accepted view in the fine arts that art is characterized by its uselessness. For Wolterstorff, works of art serve to perform a variety of actions intended to serve the artist's purpose. Egyptian tomb art, for example, was intended to (as best as we understand) accompany the deceased into the afterlife and help to provide for his/her needs and comforts. Today, religious hymns are sung by a gathering of worshipers for the purpose of praising their god.

According to Wolterstorff's approach then, mathematics—especially applied mathematics—is art at its best. Representation (numerical, graphical, and symbolic) is at its core. The worlds we project are meant to represent other worlds. Cartesian analytic geometry created a correspondence (two-way representation) between the world of Euclidean geometry and the world of algebraic equations. The differential and integral calculus has developed from a continual interplay between the world(s) of analytic geometry and the world of physical observables measured by the physicist.

Finally, this approach is clearly not Platonic, because "at its root representation is an *action* performed by human beings." We wish to emphasize mathematics as a human activity of useful representation, rather than some disembodied (abstracted from human thought and experience) realm of forms.

The aesthetic being proposed as foundational to mathematics is, of course, not an individual aesthetic, but it is a collective aesthetic of the mathematical community. Mathematics is often said to be the study of structures. However, not every structure counts as a mathematical structure. An example, as given by Steiner in [8], is chess, which has a structure, but is not considered by the general mathematical community to be mathematics. In fact, 'theorems' can be proved in chess, such as the fact that white can always win from certain positions, or the fact that a player with only a king and two knights cannot checkmate a player with only a king. Still, mathematicians generally do not consider theorems about this particular structure to be worth studying.

Historically, mathematicians often looked to utility in science as a primary criterion for a structure to be considered mathematics. Now, however, mathematicians have adopted internal criteria for making this determination. We would argue that the basic criteria for the above determination are aesthetic. G.H. Hardy, one of the great mathematicians of the past 200 years, held the view that beauty is the essence of mathematics:

> The mathematician's patterns, like the painter's or the poet's, must be beautiful; the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics…
>
> It may be very hard to define mathematical beauty, but that is just as true of beauty of any kind – we may not know quite what we mean by a beautiful poem, but that does not prevent us from recognizing one when we read it. [3, p. 85]

There are numerous guiding aesthetic principles in mathematics; some of which include symmetry, clarity, economy, unity, usefulness, representation or harmony with observed data, unexpected connections and even synergy between apparently disparate concepts, elegance in mathematical proofs and theories, beauty in mathematical patterns, and the intricate structure of mathematical systems. We claim that mathematical aesthetics has guided the course of the development of mathematics.

Some examples of places where this aesthetic foundation will be examined further in the future include, but are not limited to, the following.

First, we claim that there is an aesthetic component present in the development of the axioms from which the major corpus of mathematics is deduced. For example, when there are competing axiom sets how do we choose the axioms we choose? The reasons typically stated include such things as independence of the axioms, simplicity of the statements of the axioms, elegance, minimality of the axioms, and the elimination of paradoxes. There seems to be an especially rich aesthetic component present in the development of probability from notions of "reasonable assessments of fairness" to a fully formed axiomatic system.

Second, how does this aesthetic and theological foundation impact our view of applied mathematics? In particular, how does the concept of *mimesis* in aesthetics impact our view of what we are doing when we apply mathematics to problems in the world?

Third, there seem to be parallels in the development of mathematics to other aesthetic disciplines. These may include art, architecture, music, theater, or literature. In most of these disciplines there has been an evolution of what the community of practitioners in that discipline find aesthetically pleasing. Mathematics seems to have had a similar development, for example, from representative (counting and measuring), to realist (purely problem solving methods), to formalist (the axiomatization of mathematics), to modernist (the whole realm of mathematics can be deduced with appropriate effort), to post-modernist (Gödel incompleteness and its consequences). Some work is being done in this area, as evidenced by [2].

Fourth, this perspective of mathematics as an inherently aesthetic, creative and divinely participatory endeavor ought to inform our teaching. How do we impart to students the sense of beauty, awe and thankfulness that we find in mathematics while still giving them the foundation they need to study mathematics? Is there a difference in how we impart this sense of beauty between mathematics majors and the general student population? If so, how do we best approach each constituency? Are there projects or problems that could enhance the students' sense of awe at the beauty of the creation that is mathematics?

References:

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