

Reading Journals: Preview Assignments that Promote Student Engagement, Productive Struggle, and Ultimate Success in Undergraduate Mathematics Courses

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Abstract

We spend a lot of time searching for the *best* textbook for students. We want our students to have a reliable and useful resource to reference, as needed. We even ask them to read over certain material before classes. Often, however, we fail to guide our students in how to read that textbook productively.

Having students journal about reading their mathematics textbooks allows us to simultaneously help students struggle and persevere when encountering new problems, help students develop strategies for reading mathematical text productively, and help capitalize on what the students already have to offer. In this paper, we will look at how *Reading Journals* motivate students in a variety of mathematics courses across the undergraduate curriculum. We will further share how to develop different types of prompts for journal entries and important lessons learned.

1 Introducing *Reading Journals*

In the Fall of 2016, I taught two sections of Calculus I. Throughout that first semester, I sought to engage my students and instill in them a deeper understanding of the concepts. Thus, I wholeheartedly implemented various active learning teaching strategies. Since I was working with a new population, I thought that students would read through the material prior to coming to class. Very quickly, I suspected that very few students were ever reading the textbook and even fewer were reading before class. In the second week of classes, I recognized that I should explicitly ask who had read over the material before we worked on it as a class. When I did so, I realized that only one student from either class was reading ahead sometimes. I encouraged my students to make this a priority. I even wrote up specific *Reading* prompts in the third and fourth weeks to help guide them in their reading endeavors. Unfortunately, since these readings had no direct impact on their grades., they did not prioritize them.

In planning for the Spring semester, I still wanted active learning to be the major component of class time. Moreover, I wanted to devote the majority of class time to those parts that cause the particular students trouble rather than wasting their time going over the parts that they already knew. I believed (and still do believe) that we would be able to engage in more active learning and focus class time together more effectively if students read ahead. As a consequence, I wanted to find a more effective way to convince my students to read ahead.

During the Project NExT session *Alternative Assessment Techniques for the Active Classroom* at the Joint Mathematics Meetings in Atlanta (January 2017), Dr. David Bressoud from Macalester College shared how he had incorporated reading ahead into one of his courses. In his talk on *Assessment Practices*, Bressoud shared that his students had been required to post *Reading Reflections* one hour prior to class. The students answered various types of questions, from recognizing patterns to stating the most important point. They were always asked to describe anything that had confused them. He typically awarded credit based on completion and did not accept late submissions. When analyzing his students' final performance, Bressoud found a higher correlation between Reading Reflection scores and final grades than between mid term grades and final grades. This high correlation caught my attention and inspired me to adopt my own version. I was so encouraged by his positive experience, I decided to incorporate something similar into three of my courses that Spring. In particular, I required my students to complete *Reading Journal* assignments in Calculus II, Discrete Mathematics, and Excursions in Mathematics.

1.1 Commonalities

At Lenoir-Rhyne, Calculus II, Discrete Mathematics, and Excursions in Mathematics are all 100-level courses. As such, one of my main goals was for students to **struggle, make mistakes, and persevere**. This student outcome was a major driving force behind the motivation for, and the design of, the *Reading Journal* assignments. In each syllabus, I included the following description of *Reading Journals*:

This allows you to take on a more active role in learning about the new material. Further, your solutions provide us with the invaluable opportunity to note where your understanding is strong already and where your understanding can grow further. You will have a number of opportunities to share your work from your reading journals during class.

I wanted my students to read over specific material before we covered it in class. I also wanted my students to be resourceful in using their textbooks as a reference. Therefore, I required my students to read through upcoming material. Like Bressoud, I also asked my students to respond to specific questions from the material that they had read. Further, I graded the *Reading Journal* assignments based on completion. Unlike Bressoud, I was not available to read my students' responses the hour before class. Therefore, my *Reading Journal* assignments were due at the start of class. My students were expected to share their solutions at this time. While they were engaging in initial discussion of their solutions with partners, I was able to verify whether each student had completed their assignment. To encourage students to take these assignments more seriously, the *Reading Journal* assignments accounted for 10% of their final grades. In summary, these assignments followed the following basic format in all three courses.

- Students read through the material in the upcoming section or chapter. While reading, students were expected to:
 - compile notes from reading; and
 - attempt the assigned problems.
- Any time the student was unsure of how to proceed in a particular problem, the student was expected to:

- read over the examples and detailed solutions provided in the text;
 - mark any conflicts or differences; and
 - write down questions as they arise.
- Students were expected to share their solutions and/or responses at the start of class.
 - As a class, we went over the shared solutions. We added any missing details and addressed all of the content questions students posed with the full group.

1.2 Course Specific Information

As mentioned above, the underlying motivation and structure of *Reading Journal* assignments were the same for Calculus II, Discrete Mathematics, and Excursions in Mathematics. Yet the content and, subsequently, textbooks were so different in these three undergraduate courses that the *Reading Journal* assignments for each class had its own style. As such, we will elaborate on the remaining student learning outcomes and textbook for each course. Then we will detail the particularities of the assignments and how they were incorporated into the corresponding class.

Calculus II

In this second course of calculus, our students study integration, sequences and series, applications of the integral, and inverse functions. In addition to struggling productively, students who successfully completed my course this past Spring should have been able to:

- Understand, and apply, integration techniques to determine areas of regions, volumes of solids, and lengths of arcs;
- Understand, and be able to apply, various techniques of integration to determine the antiderivatives of a wide variety of functions in various situations;
- Understand infinite sequences and series, discuss the convergence or divergence of certain series, and use Taylor polynomials and Taylor series to approximate functions and functional values;
- *Understand and successfully apply various problem solving strategies, when appropriate;
- *Communicate about, and analyze, various problem solving strategies;
- *Analyze and clearly present multiple solutions to a given problem, both orally and in written form; and
- *Understand, and analyze, multiple ways to learn.

The *Reading Journal* assignments were designed to encourage students to meet all these goals, especially the ones highlighted by an * not specifically about content. We used *Calculus Early Transcendentals*, (Sixth Edition), by James Stewart. Each section of this book provides conceptual explanations along with several problems and their solutions. The student learning outcomes together with the structure of our textbook led me to use this additional structure for Calculus II *Reading Journal* prompts:

- I assigned 2 to 3 problems, *whose solutions were provided*, from the upcoming material at least one day before our class meeting in our online learning management system; and
- In class, students were expected to write up their solutions on the board.

Discrete Math I

In our first semester of Discrete Mathematics, students gain exposure to, and an appreciation for, various mathematical applications to computer science. Some of the topics include the nature of mathematics, basic logic, lists and sets, relations and partitions, functions and their properties, graph theory, and discrete probability theory. Beyond developing productive struggle strategies, students who successfully completed my course this past Spring should have been able to:

- Apply basic counting and probability techniques to solve specific problems;
- Apply proofing techniques, such as induction, to prove simple mathematical statements;
- Construct truth tables and apply deductive reasoning;
- Calculate basic sums and/or products;
- Verify certain properties of a given relation, such as reflexivity, transitivity, *etc.*;
- Apply certain basic techniques from Graph Theory and Trees to solve specific problems; and
- *Be familiar with, and communicate about, some skills and knowledge that can be applied in Computer Science.

In addition to mastering the content specific student outcomes, my students were challenged to develop their endurance in problem solving and their ability to communicate effectively. The *Reading Journal* assignments were designed not only to lay a stronger foundation for learning new content but also to encourage students to pursue these latter two goals. The *Reading Journal* questions were to be completed after reading through appropriate sections in our text, *Mathematics: A Discrete Introduction*, (Third Edition), by Edward R. Scheinerman. This book provides definitions followed by several examples and a few highlighted notes. The student learning outcomes together with the structure of our textbook led me to use the following structure specifically for Discrete Mathematics *Reading Journal* prompts:

- I posted a few questions, *whose solutions were not given*, spanning the upcoming section at least one day before our class meeting in our online learning management system;
- In class, students were expected to share their responses with one or two people sitting next to them. During this sharing, students had an opportunity to get feedback from peers and myself;
- Then a couple of students shared their responses with the entire class. I wrote up the information as they shared; and
- We discussed the rest of the material through the lens of these student responses.

Excursions in Mathematics

Excursions in Mathematics is a three hour course for students who need a mathematics course but do not have to take Calculus. To simultaneously expose my students to various mathematical concepts and encourage them to develop an appreciation for mathematics, I focused on developing various problem solving strategies throughout the course of the semester. Students who successfully completed my course this past Spring should have been able to not only struggle productively but also:

- Understand and successfully apply various problem solving strategies, when appropriate;
- *Communicate about, and analyze, various problem solving strategies;
- *Analyze and clearly present multiple solutions to a given problem, both orally and in written form; and
- Argue about the significance of having access to various problem solving strategies.

In this course, the *Reading Journal* assignments were designed to help students tackle the above outcomes which are emphasized by an *. They were meant to be completed as students read a particular chapter of *Crossing the River with Dogs: Problem Solving for College Students*, (Second Edition), by Ken Johnson, Ted Herr, Judy Kysh. Each chapter of this book focuses on one problem solving strategy. As you read a given chapter, you encounter a few problems. As the reader, you are expected to attempt these problems before continuing to read. Following each problem, the authors share at least one actual student solution and sometimes even discuss it. To reinforce the goals of this text, I incorporated these problems into the course by having students tackle them for their Excursions in Mathematics *Reading Journals*:

- Every time the text introduces a new problem, students were expected to attempt to solve that problem using the current problem solving strategy;
- Any time the student was unsure of how to continue solving a particular problem, *which was followed by at least one solution*, the student was expected to keep reading; and
- Students were expected to present their own solutions, offering explanation and answering questions from their peers and myself.

In addition to the 10% of their grade coming from these regular *Reading Journal* write ups and presentations, students were asked to write a reflection of their personal development as a problem solver throughout the course; in particular, as they developed their skills through the process of attempting to answer the *Reading Journal* problems. This assignment counted an additional 5% towards their final grades.

2 Developing Prompts

2.1 Calculus II

There are a number of problems together with solutions in every section of this text. One could assign students to work through and understand the solutions to all of the problems provided in a given section. Since there are so many problems, though, I opted to carefully select about two or three problems that were a representative sample of all of the ones provided. The representative example problems often served as the question prompts, as the following six prompts indicate.

- Read over section 5.5 *The Substitution Rule*.

Work out Example 3, Example 5, and Example 9 from the reading.

*Yes, these examples are all worked out in your text. Try to work them out without looking and then compare! Some of you will present your solution to the class (with your own notes and without using your book).

- Exam I Review Reading Journal

Recall that we have covered Chapter 5 *Integrals*, 6.1 *Areas Between Curves*, 6.2 *Volumes*, and 6.3 *Volumes by Cylindrical Shells*. As you are reviewing for your first exam, think about different types of problems.

- i. Create two of your own problems that address two different topics we have covered. This exercise is meant to help you review further.
- ii. Attempt to set up, and solve, each of your own problems.

- Read over section 7.3 *Trigonometric Substitution* pages 467 through 472. As you read, work through Example 3, Example 6, and Example 7 on your own.

Make note of any questions that arise!

- Section 7.4 RJ Part I

Read over section 7.4 *Integration of Rational Functions by Partial Fractions* pages 473 through 481. As you read, rewrite the integral as the integral of a sum of partial fractions in Example 2, Example 4, and Example 5 on your own. Do not evaluate the integral.

Make note of any questions that arise!

- Reread section 7.5 *Strategy for Integration* pages 483 through 488. As you read, write out the method of attack for Example 1, Example 2, Example 3, Example 4 and Example 5 on your own.

You do not have to fully work out each integral.

Make note of any questions that arise! Are there any you would like to work out as a class?

- Section 11.8 *Power Series* RJ 2

Complete problems 21 and 29 on page 727. You should at least write down the problems and attempt to solve them.

Make note of any questions that arise!

Please, bring homework questions.

2.2 Discrete Mathematics

My Discrete Mathematics *Reading Journal* prompts took the longest to create. Rather than borrowing problems from the actual reading, I developed my own prompts. Here are ten of my *Reading Journal* prompts presented in the order in which we discussed them as a class.

- A. Read *To the Student* on pages xvii-xviii.
 - i. In your own words, explain the difference between continuous mathematics and discrete mathematics.
 - ii. Include an example.
- B. Read Section 4 *Theorem*.
 - (a) Create your own A and B so that the statement “If A, then B.” is true.
 - (b) Write out the following statements using your A and B. For each statement, determine whether it is true, sometimes true, or false.
 - 1. “If not A, then not B.”
 - 2. “If B, then A.”
 - 3. “If not B, then not A.”
- A. Read Section 4 *Theorem*.
 - i. In your own words, explain the difference between nonsensical statements and vacuous statements.
 - ii. Write your own nonsensical statement.
 - iii. Write your own vacuous statement.
- B. Read Section 7 *Boolean Algebra*.
 - i. Write out a program to evaluate Boolean expressions. Your program should test if these expressions are tautologies, test if two expressions are logically equivalent, etc.
- A. Read Section 8 *Lists*.
 - i. The author discusses two different types of lists and how to count them.
 - (a) Create your own example of each type and
 - (b) highlight their differences.
 - ii. Attempt problem 8.16 on page 39 in your book. (You should at least write the problem down and draw boxes.)
- A. Read Section 10 *Sets I: Introduction, Subsets*.
 - i. In your own words, explain the difference between \subseteq and \in .
 - ii. Create your own set.
 - a. Identify at least two different elements of your set and
 - b. at least two different subsets of your set.

Notice that this restricts the possible sets you may consider.
- A. Read Section 14 *Relations*.

Pay close attention to the definition of a relation and the unpacking of the different properties of relations. In the past, I have generated questions to encourage close reading and sufficient

understanding of the text. Studies show that reading students develop a deeper understanding when they formulate their own questions over the reading. Thus, for this reading journal, you are going to take on the role of instructor.

- i. What Would You Ask? - Develop two to three questions you would ask the class, as the instructor, to check their understanding of a particular relation. Your questions should check for a deeper understanding of the material.

Note: These are questions to examine the understanding of your peers. You may or may not know the answer yourself before class.

*You will be given a particular relation in twos or threes. Then each of you will ask your fellow group member(s) the questions you have created. As a group, your questions will be shared and discussed.

- Read Section 24 *Functions*. Notice that there are definitions for function and inverse function.
 1. Create your own function whose inverse relation is not a function.
 2. Create your own function whose inverse relation is a function.
 3. Write your own question over the material in this section.
- Carefully read Section 5 *Proof*.
 1. Prove that the sum of two odd integers is even. (Propositions 5.2 and 5.5 will be extremely helpful here).
 2. Describe the two proof templates presented in this section. Write down specific steps associated with each.
- Carefully read Sections 30 *Sample Space* and 31 *Events*.
 1. The outcomes of an experiment are the elements of the sample space. Suppose you are rolling two dice. List all the possible outcomes. (There are 36, not 11, altogether!)
 2. Create your own sample space. Determine what the associated outcomes and events are.
 3. In your own words, describe the difference between an outcome and an event.
- Carefully read Section 53 *Planar Graphs*.
 1. Sketch your own (a) simple curve and (b) simple closed curve.
 2. Consider the crossing-free drawing of a connected planar graph on the top of page 369.
 - (a) Make 4 other crossing-free drawings of connected planar graphs.
 - (b) For each of those 4 graphs, record how many vertices, edges, and faces the drawing has.
- Carefully read Sections 49 *Connection* and 50 *Trees*. Consider the graph

$$G = (\{a, b, c, d, e, f, g\}, \{ab, cd, de, df, dg, eg\}).$$

1. Sketch G .
2. Identify a walk in G that is not also a path.
3. Identify the components of G .
4. Identify at least one cut vertex of G .
5. Identify at least one cut edge of G .

6. Identify a longest cycle in G .
7. Identify a spanning subgraph F of G that is a forest. Sketch F .
8. Determine how many leaves F has.

As these different prompts illustrate, I was intentional about asking different types of questions in their *Reading Journal* prompts. Students were challenged to answer specific prompts, explain a new phenomena in their own words, generate and study their own examples, and/or generate their own questions over the new material. Occasionally, I would even ask students to complete one or two of the problems at the end of the section. I was conscientious about writing prompts that spanned as much of the section as possible.

2.3 Excursions in Mathematics

The book *Crossing the River with Dogs: Problem Solving for College Students* is set up for interactive reading. The authors intend for students to read the material prior to coming class. Moreover, as it says in the introduction, the authors want students to struggle and make mistakes as they attempt the problems in the reading. Then the students are meant to check for mistakes and persevere as they continue reading the text and, subsequently, encounter alternate solutions. Thus, this text lends itself most naturally to *Reading Journal* assignments which foster productive struggle. I have included four specific *Reading Journal* prompts below.

- Read Chapter 1. Attempt the problems as they arise in your reading. There are four problems.
- (The class was evenly divided between Chapters 5, 6, 7, 8, and 9 for this *Reading Journal*.)
 1. Read your assigned chapter.
 2. Take notes of the main ideas.
 3. Write down questions that arise.
 4. Completely work out two of the problems discussed in the text of your chapter.
- Under announcements, you will find “Geometry Unit 4.4 Reading and Exercises”.
You are responsible for reading the attached file *GeometryUnit4.4_ReadingJournal.pdf* before class tomorrow. Complete the exercises as you read. Bring your work and any questions to class.
- (1) Carefully read Chapter 17 Section 1. (469-484)
 - (2) Attempt the problems as they arise in your reading.
 - (3) Take note of how the problems are solved in the reading.
 - (4) Create your own questions. - Develop 2 to 3 questions over the material from Chapter 17 Section 1.

As these four representative *Reading Journal* prompts indicate, the question prompts are the problems included in the chapters already and the assignment descriptions became more thorough as the semester progressed. As stated above, students were always challenged to use a particular problem solving strategy in their work. As in Discrete Mathematics, I sometimes required the students to generate their own questions.

3 Lessons Learned

The first time we implement a new teaching strategy in the classroom, teachers often learn so much about how to make that new strategy even better the next time they use it. Not surprisingly, I learned a lot about incorporating *Reading Journal* assignments into different classes. While reflecting on my experience and feedback from students (both informally and formally), I believe that there are several changes that can be made in future *Reading Journal* assignments to help students value them more and to make them more beneficial to students.

There were two common themes that were most prominent in all three courses. First, it is important to support students in developing resourcefulness. Second, it is crucial that students understand what is expected of them and why. Answering these questions once is not enough. Students need to hear the what and why repeatedly. Looking back, I wish we had spent more time emphasizing what the purpose of the *Reading Journals* was and why I was having students complete them. By not emphasizing these ideas enough, I invited the students to draw their own conclusions. While I was excited that students were frustrated and had hit a wall in their understanding, the students were upset and felt like they had been abandoned to figure everything out solo. By more openly discussing these feelings, I hope to help students focus on overcoming the roadblocks rather than the roadblocks themselves.

3.1 Calculus II

Almost all of the Calculus II *Reading Journal* prompts were exercises from the text. Therefore, students had access to a full solution right away. This immediate feedback allowed students to build up their confidence before volunteering to share their own solution with the entire class. I had hoped that students would spend a reasonable amount of time working on a problem before referring to the solution provided. In practice, though, a majority of students simply copied down the solutions provided in the book. Although copying does not tell us as much information as honestly trying a problem before comparing our work to others' solutions, I had hoped the process of writing the solutions down would force students to actually think about the material.

However, in an anonymous survey at the conclusion of the semester, several students shared that one of the most frustrating aspects of the *Reading Journal* assignments was that they were so easily able to copy down the work without giving the problems and ideas enough thought. Perhaps this frustration explains why students' final grades did not correlate uniformly with their *Reading Journal* scores. Most students recognized that *Reading Journal* assignments were 'easy' points, so they wrote them up late at night, often only copying. I remember students quickly wrote up solutions to the questions from *Reading Journals* 5.5 and 7.4 above and asked little to no questions. Since they had not struggled enough with the material before class, they did not know which pieces they did not understand or where they were stuck. On days when students did not come to class with questions, the majority of students did not even recognize that we were going over the material. When asked to give advice, my students warned future students to start working on *Reading Journals* early enough to think about them. They further advised students to actually write down questions as they worked. While many students did admit to copying without paying attention, they did recognize that a more focused attempt would be more beneficial. In addition to sharing pointers, I need to lead discussions about the importance of finding where we struggle most in problems.

Sometimes these assignments did help students recognize where their understanding was weaker and where they had lingering questions. Not surprisingly, our most memorable class discussions originated from students asking questions about particular steps and confusing transitions. For example, students wanted to understand why the authors had chosen a particular trigonometric substitution in the examples from 7.3. This allowed us to work together to carefully analyze the differences in those problems. To address questions raised about 7.5, we discussed why the authors made certain choices and how to set up the integral for a given technique. That day, students had thought about which problems they most wanted to go over together. In retrospect, I think having students articulate why they saw certain problems as more challenging than other problems would have led to an even more fruitful conversation about where different people get stuck and strategies for overcoming these hurdles. Students constantly expect immediate feedback and ask for step-by-step instructions while working, independently or otherwise, on problems. Since *Reading Journals* seem to go against these perceived needs, it is important that we spend more time discussing productive struggle and its value with our students.

Unlike for most *Reading Journals*, students could not copy solutions from the text for the Exam 1 Review and 11.8 Part 2. These assignments pushed students to use their understanding of material to write their own solutions. I still wanted students to identify key ideas and figure out where their understanding needed refinement. Students reported that they found it easier to write their own questions than to write their own solutions. I used their sticky points to make the review more meaningful. Once more, it would have been extremely helpful to explicitly bring this fact to their attention. These prompts were challenging students and causing them to struggle. In the future, I want to spend more time discussing how we persevere in those moments.

Although students often had access to full solutions, a couple of students did not regularly attempt to solve these exercises. They did not even copy down the solutions in the book. Rather than reaching out for help, they gave up. Some students admitted to feeling under prepared to present new material. Yet most students completed their *Reading Journal* assignments regularly. Sometimes students would explicitly express that they had experienced frustration with a particular concept while completing the problems and then felt much better about these concepts after discussing them in class. By having students practice some problems before class, we actually spent most of our time discussing the concepts. Rather than wasting time going over the parts that the class already understood, we spent the majority of our time working out the finer details together. As mentioned above, students had trouble recognizing what we were doing and why. By allowing students to struggle and make mistakes before class, we were able to persevere and develop a deeper understanding altogether. Since it led to a richer and more focused discussion, I viewed the student frustration as fruitful. Unfortunately, a number of students let their frustration bring them down. Two challenges are selling the frustration feature in the context of productive struggle and helping students understand what it looks like to spend a reasonable amount of time working on the material before class.

3.2 Discrete Mathematics

Recall that I created the *Reading Journal* prompts for Discrete Mathematics based on the assigned readings. It is important to stress that these questions were not taken directly from the text. However, they were still designed to check student understanding of the material before class discussions. Since students were not able to copy down solutions from their textbook, they were challenged to think more deeply about the material. The students who were more willing to engage with material before class

tended to perform better overall. Thus, it makes sense that the correlation between *Reading Journal* scores and final course grades was more linear in Discrete Mathematics than in Calculus II. As in Calculus II, some students in Discrete Mathematics expressed feeling frustrated while completing their *Reading Journals*. Unlike in Calculus II, students did not have immediate feedback for the problems assigned. Subsequently, some students expressed feeling completely lost while reading the text. In general, though, students were more comfortable creating their own examples than analyzing them. Sometimes students did not feel like they had enough understanding to even attempt to create their own examples. In spite of our discussions about expectations for making an honest effort and first steps in making an attempt, some students still reported discomfort in working on material before discussing it in class. During the semester and in their survey, students were most consistently vocal about their dislike in creating their own questions. One important change is to better equip students with tools to persevere through their feelings of frustration and being lost. Another big challenge is helping students internalize that I am looking for completion, not accuracy.

When students felt better about their *Reading Journal* responses, more students actually spoke up sooner in our full class discussions. Perhaps as expected, students preferred creating their own examples over attempting to analyze them. During class, many students were willing to share their own examples while only a handful of them would share their analysis of these examples with the entire class. For instance, we developed a more thorough understanding of the two types of mathematics by sharing our individual ideas for part A of the first prompt included above. When discussing part B of that same prompt, a large number of students shared their initial statements for (a). Of these students, only a couple were willing to share their responses for (b). As a class, we analyzed these statements, verified whether they were correct or not, and determined whether they were true or false. Students had struggled when writing their own statements and determining whether they were true on their own. Then we were able to uncover the subtleties together. In the future, I want to emphasize how working together is one valuable way to overcome our frustrations more consistently.

Throughout the semester, students repeatedly expressed their frustrations from not immediately knowing whether they were correct or incorrect in their responses. During our second exploration of Section 4 as a class, a few students shared their statements and which type they had labeled them. When we analyzed them further, we were able to pinpoint misconceptions resulting from the seemingly subtle differences between the two types of statements. We encountered a similar situation with Section 10. We first discussed a couple of their more straightforward examples together. Then we struggled through a more interesting example all together. As a result of grappling with this harder example, students were better able to internalize the difference between elements and subsets. When comparing answers to Sections 30 and 31, I was overjoyed to see the students who only found eleven outcomes realize which outcomes were missing from their list. Once more, the students who make a mistake are more likely to learn the material when correcting that mistake. The different responses led to a more productive discussion about the importance of knowing whether we distinguish between objects or not. While I was excited that we so often debated ideas and overcame misconceptions together, a number of students were annoyed. A number of times, I verbally expressed that I was happy that they had struggled with these ideas. I also pointed out how working together allowed us to all have a better understanding of the ideas. By resolving disagreements, we are better able to remember what we have learned. Even after sharing these benefits with the class, some students were still not happy. A few students were so frustrated at making mistakes initially that they were not able to appreciate how much more they learn this way.

Usually students were happy to generate their own examples. However, there were times when students

felt ill equipped to do so. In particular, a number of students struggled to create their own examples for Sections 24 and 53. During class, they even complained about not having enough support to actually create functions satisfying the given restrictions and sketches satisfying the given conditions, respectively. Luckily, in these situations, peers would often point out how helpful it was to consider the examples from the assigned reading. Sometimes it was even possible to choose one of the examples from the text. In the future, I would like to spend more time discussing how failing to create functions and/or sketches actually fuels a richer discussion. Moreover, overcoming mistakes allows us to develop a better understanding of those concepts and, subsequently, remember them.

More generally, a few students were uncomfortable working through problems before we discussed the material as a class. Sections 8 and 5 were particularly challenging for students. For Section 8, they completed one of the exercises immediately following the reading. In the past, some students had voiced concerns about not knowing how to start a response. We had discussed the value in actually writing down the problem. As a reminder, I gave them the first step in the prompt. Together we discussed how far everyone had come in solving the problem. We shared strategies for moving from the problem statement to actually solving the problem. For Section 5, I included a hint to focus on trying to use specific propositions in the hopes that students would at least write down these propositions. Since writing proofs is all about communicating in a logical way as effectively as possible, this concept proved to be extremely challenging. When students voiced their frustrations with this *Reading Journal*, I emphasized the importance of using the context clues and all the given information. We explicitly discussed the fact that I expected everyone to write down the statement to be proved and the two propositions from the hint. Then we would be ready to untangle all of the ideas together. While such conversations were helpful in the moment, some students had more trouble applying the ideas to new situations. Thus, it is important to have these conversations more often. In the future, I want to further discuss the importance of not giving up on ourselves when we cannot completely answer something.

Overall, students were most vocal about not wanting *Reading Journal* prompts which required them to generate their own questions, as with Section 14. This surprised me. To me, generating questions is akin to generating examples. However, a majority of students struggled with wanting to completely understand and answer correctly. While I wanted students to struggle and uncover where their understanding was weakest, students wanted to show that they were correct. As a result, students were uncomfortable writing down questions for which they did not already have valid solutions. Switching the focus from finding complete valid solutions to giving an honest effort was a constant struggle in this class. While we did address this struggle together, some students were still not able to break that habit. Like 14, Section 49 frustrated the students. They wanted to be able to answer everything correctly right away. Next time, I would lead a discussion on how to initiate the solution to a given problem without stressing out about whether we can completely do so.

The most memorable *Reading Journal* prompts required students to create and analyze their own examples of a particular concept. Students were often more willing to share their own examples with the class. Moreover, students seemed more engaged in the material when they created their own examples. Focusing on creating an example rather than answering a question correctly helped more students be okay making mistakes. Whenever students created their own examples, we were able to discuss the entire section through further analysis of their examples. The conversations centering on student-generated examples encouraged much more animated conversation from the class. In addition to being more interesting to our class, these examples encouraged deeper conversations as we sought to analyze them. Their examples did not necessarily have one correct answer. In fact, there were often more possibilities. Thus, we worked more on developing deeper reasoning skills. Therefore, I felt that these *Reading*

Journal prompts worked exceptionally well for our class.

3.3 Excursions in Mathematics

Due to the nature of the text, students were able to see alternate solutions immediately after working on a problem. Thus, they were able to receive instant feedback on their understanding of the question and its solution in the context of a particular problem solving strategy. This access to full solutions allowed them to feel better about the parts that they had done correctly. The text also allowed them to compare their own write up to a possible solution and to see where their write up differed. While I had hoped that everyone would share different approaches with the class, few students did so in practice. However, student feedback on the anonymous survey and in their final reflection on these assignments was very insightful. The Reflection assignment asked students to choose three to five problems from different reading journals to submit with a reflection on how their ability to solve problems throughout the semester evolved and how reading journals contributed to helping them learn to solve problems successfully. Overall, students in this course were more willing to take advantage of the opportunity to struggle productively in and out of class than students in the other two courses.

The students who completed more *Reading Journals* tended to perform much better in the course overall. While I did check for physical evidence, it was easy to note which students were spending more time and energy on completing the *Reading Journal* problems through the presentations and our full class follow-up discussions. The end of semester survey completed by half the class further revealed that students spending less time on these assignments were more negative about them. They did not feel that the *Reading Journals* were beneficial and some even stopped doing them completely. On the other hand, students who spent more time on these assignments reported much more positive feelings about them. When asked for advice, they encouraged future students to take these assignments seriously. They shared how beneficial it was to really think through the problems and compare to the solutions provided.

Students who had spent less time with the material had to focus on the big picture during class and struggled to figure out the more subtle details on their own later. For *Reading Journals* like for Chapter 1, they had little to work down for the different problems. They rarely wanted to share a solution with the class. They tended not to pose, nor answer, questions. When students were responsible for highlighting main points for an assigned Chapter, as in the assignment for Chapters 5, 6, 7, 8, and 9, these students contributed less to their small group discussions and were not as comfortable sharing the work with other students during the speed interview discussions. They also tended not to have any questions written down when coming to class. For example, they were not as likely to have questions for the Geometry text of Chapter 17. Their final reflections emphasized that they did not find it beneficial to see the solutions in the text. One student even admitted to not doing them, because there was not value in doing so. They did not see the value in struggling productively. As expected, they reported that they did not perceive any growth in the ability to solve problems.

The students who spent more time digesting the material before class were better equipped to deepen their understanding during class. They had a better understanding of what they knew and where they had lingering questions. Since they had spent time with the material already, they were able to focus more on the nuances. In their reflections, these students highlighted how valuable it was to really think about a problem and its solution. They recognized how helpful it was to read about a problem solving strategy and work through some problems before talking all together. They appreciated the opportunity

to check their work with the step by step solutions provided. One student even stated that every math class should use *Reading Journal* assignments so that students can look over what is coming next and have the opportunity to ask questions that allow them to deepen their understanding even more. As the semester progressed, one student mentioned that (s)he started thinking about each problem in multiple ways and tried to choose the most efficient solution. They saw the value in struggling to tackle various problems using a new technique. Further, they were able to persevere in their journals. They continued to persevere by developing an even better understanding when we went over problems as a class.

As in the other two courses, the students who fell between the low and high achievers tended to want my help and immediate feedback while working on these assignments. They also felt like they would have benefited more from me presenting a correct solution than a peer presenting their own solution. While I did address that there is far greater value in learning from each other and mistakes, my message did not seep in with everyone. In the future, I plan to more regularly ask who got stuck while working and how they got unstuck or how we will get unstuck together. Unlike the other courses, there was a stronger correlation between students who completed *Reading Journals* and students who performed well overall. Therefore, one of the biggest challenges in Excursions for Mathematics is getting the students to spend time completing the *Reading Journals*.

4 Looking to the Future

4.1 Calculus II

If I expect students to read their math textbooks productively, I have a responsibility to model what this looks like. When we are reviewing some integration concepts from Calculus I, I would like to find a way to intentionally model how to read the textbook productively. I also want to be conscientious about asking the class how many people struggled with the reading and how many people had felt frustrated. I want to follow up by complimenting them for doing so and reminding them of the value of getting stuck and struggling productively.

4.2 Discrete Mathematics

As in Calculus II, I feel that it is crucial to better prepare students for reading mathematical texts productively. In addition to modeling how to read, I want to generate an ongoing list of tools and resources available to us when we feel stuck or lost. On a regular basis, I want to recognize that we should all feel lost at different points when reading. I want to regularly emphasize that where we get stuck is where we most need help. I want to be conscientious about reminding students that the places we get stuck on our own are the places we need to focus on together. So the purpose of completing *Reading Journals* is not to have everything correct. The real purpose of these *Reading Journals* is to guide us to what we need to spend the majority of our time together discussing. In this way, we work together to struggle even more productively.

4.3 Excursions in Mathematics

Although we would spend at least two days discussing a new chapter, I had students read an entire chapter and complete all of the problems in that chapter before our first meeting. My students would benefit from a more regular schedule and more focused *Reading Journal* assignments. Thus, I would like to split the *Reading Journal* assignments for each Chapter into two parts. I hope that this change would encourage students to spend a little more time reading each part. Since there would be fewer problems due each day, I also hope that they will spend a little more time tackling those problems. I think that this would alleviate pressure on the students and support a more focused conversation each day. Another consequence is encouraging us to more regularly discuss strategies for making mistakes and learning from those mistakes in a productive way.

5 Closing Remarks

As I mention above, one of my main goals in teaching is to help students struggle, make mistakes, and persevere. Many of my students tend to fear making mistakes and are reluctant to try tackling a problem before seeing an expert tackle a few problems of that same type. Rather than giving in and doing what my students are most comfortable with, I push my students to take more ownership of their learning. I am driven by my desire to equip my students to develop a stronger foundational understanding of mathematics.

Since we spend so much time and energy finding the best possible resources for our students, why would we not find productive ways to get them to use those resources? *Reading Journal* assignments not only encourage students to actually read their textbooks but also help direct the focus of our class time to those concepts that cause students to struggle the most. These are the places we most need tools for persevering and would most benefit from discussing together. They allow students to recall the information they already know and to figure out what new information makes sense to them. These assignments also show students where they are confused and frustrated. These latter parts are the ones we should be discussing in class. *Reading Journal* assignments help students and teachers find out where to spend the most time. Further, they allow us to practice struggling productively.