Finding Meaning in Calculus (and Life)

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Abstract

The 2015 publication, *Insights and Recommendations from the MAA National Study of College Calculus*, noted that "students taking college calculus exhibited a reduction in positive attitude toward mathematics, which can affect their career aspirations and desire to take more mathematics". The study concluded that students' confidence to do mathematics, enjoyment of mathematics, and desire to persist in their study of mathematics had all decreased by the end of their college calculus experience. This paper will summarize the findings of the MAA's study of college calculus and then suggest how a Christian foundation can prove to be beneficial in improving student attitude toward calculus and mathematics in general.

I have seen all the things that are done under the sun; all of them are meaningless, a chasing after the wind. Ecclesiastes 1:14 (NIV)

1 Introduction

I have been teaching college calculus for more than 30 years now. Not only do I enjoy teaching the subject, I enjoy the material itself. Moreover, I believe there is value in studying calculus. I believe that those who master the subject will be better equipped to think about and understand God's creation. As a teacher, one of my greatest desires is to instill that same appreciation and understanding of calculus into my students. Unfortunately, many of my students do not share my affinity for the subject. Some struggle with course content from the onset of the course while others see little or no connection to their future careers. Apparently, my experience is not unique. The Mathematical Association of America's (MAA) recent study of college calculus students concluded that "students taking college calculus exhibited a reduction in positive attitude toward mathematics, which can affect their career aspirations and desire to take more mathematics" [3].

What leads to a reduction in positive attitude toward mathematics among students who take college calculus? The aforementioned study mentions one potential explanation for this reduction in attitude, namely increased rigor when compared with pre-college work. It is important to note that reduction in student attitude is a result of increased rigor and not just rigor alone. After years of training at the secondary level, it is natural for students to have developed some expectation of what their college mathematics experience will entail. Research shows that expectation and potential value derived are

key factors when it comes to achieving learning objectives [1], so it should not be a surprise when students who begin their study of calculus with unrealistic expectations experience a reduction in positive attitude.

As mentioned above, one of my objectives as a teacher of calculus is to help students find meaning in their study of the subject matter. Because achieving this objective is closely related to my students' values and expectations, it is crucial to identify and understand these values and expectations. Therefore, my purpose in this article is two-fold. First, in order to better understand the values and expectations of college calculus students, I will summarize the context surrounding these students as they prepare to enroll in Calculus 1. Second, I will present some thoughts on helping students think about what they value and how their value systems may be perpetuating gaps in their understanding and also leading to a lack of meaning in their education. I will share my experience during the past thirty years of teaching college calculus, as well as some of the findings of the MAA's national study of college calculus. It is important to keep two things in mind as you read this article. First, my focus is on Calculus 1 as a college course. That is, I am not considering the calculus sequence as a whole. Second, the thoughts proposed in this article rely heavily on Christian values and as such any implementation of these ideas is likely limited to the Christian college classroom.

2 Setting the College Calculus 1 Stage

According to the MAA's study of college calculus, 500,000 students enroll in a college Calculus 1 course each year. About half of these students are seeing calculus for the first time. My goal in this section is to get a general sense of who these students are. What was their high school mathematics experience like? Are they prepared to take college calculus? What are their expectations? I make no claim to be exhaustive here. I rely heavily on the 2015 publication, *Insights and Recommendations from the MAA National Study of College Calculus* [3] for most of my answers and suggest it as a resource for the reader who is interested in more extensive answers to the questions posed above and other questions like them.

Much has changed during my 30 years of teaching college calculus. Perhaps the biggest change has been in the area of technology. Graphing calculators were unheard of when I studied calculus back in the early 1980's. I owned and used a calculator while learning calculus, but only for difficult numeric computations. In contrast, my students can do most of what I try to teach them just by punching a few buttons on their calculators. According to David Bressoud, author of the introductory chapter in the MAA's report on college calculus, "permission to use graphing calculators on exams is one of the sharp discontinuities between high school and college calculus" [3]. In the report, Bressoud notes that graphing calculators are almost universally used in high schools. This creates a potential problem for students who enroll in a college Calculus 1 course for which the use of a graphing calculator is either restricted or not allowed at all.

Some of the changes that have occurred during the past few decades are a direct consequence of students' struggles with the content and relevancy of the subject matter. Nearly 35 years ago, Robert White (President of the National Academy of Engineering) suggested that it was time to turn calculus from a filter to a pump, a reference to calculus as the reason some aspiring engineers were leaving the major. The calculus reform movement was born shortly after this statement was made (about the time I was beginning my teaching career). During my thirty-plus years of teaching, I have attended many talks about new and innovative ways to teach mathematics. My bookshelves are filled with textbooks empha-

sizing different approaches to learning calculus. Terms like student-centered learning and the flipped classroom, which were not part of my educational experience, are now very familiar to me, and I have experimented with some of these approaches in my classroom.

Have the changes mentioned above impacted student attitude toward calculus? According to the MAA's report, educational technology "was found to have no impact on attitude", while ambitious teaching "had a small negative impact on student attitudes". In contrast, the MAA's report noted that "instructors who employed generally accepted good teaching practices (e.g., clarity in presentation and answering questions, useful homework, fair exams, help outside of class) were found to have the most positive impact, particularly with students who began with weaker initial mathematics attitudes" [3]. Despite all the advances in technology and innovative approaches to teaching that have been made during the past three decades, Bressoud opened the MAA's report by admitting that calculus reform has made little progress, and calculus in many ways still remains a filter.

In order to understand why college calculus has remained a filter, it is not enough to only consider a student's experience in the college classroom. More than three-quarters of all students who study calculus have their first experience with calculus in high school, and as previously mentioned, approximately half of all students enrolled in a first-semester college calculus course have already studied calculus in high school. In discussing the impact that a high school calculus course can have on a college student, David Bressoud makes the following observation, "Mathematics is unique among all disciplines in having created a course, calculus, which is both the lodestar of the K-12 curriculum and the bedrock of post-secondary preparation for science and engineering. These distinct perspectives on this course create much of the discontinuity that students experience as they transition from high school to college" [3].

It has become virtually impossible to talk about the transition from high school to college mathematics without considering the impact of the AP exam. In 1989, 79,000 students took the AP exam [2], while in 2010 more than 400,000 United States students took the exam [3]. For some students, a good score on the Calculus AP exam exempts them from taking any mathematics classes in college, while for others it serves as a jump-start to their college education. According to Bressoud, the problem with jump-starting a college mathematics curriculum using the AP exam is that success in AP Calculus does not require the level of proficiency in mathematics that post-secondary faculty want their students to have when it comes to their mathematics, science, and engineering courses. He adds, "In some sense, the worst preparation a student heading toward a career in science or engineering could receive is one that rushes toward accumulation of problem-solving abilities in calculus while short-changing the broader preparation needed for success beyond calculus" [3].

The impact of the AP exam is especially felt in a college Calculus 1 class. That might seem like a strange statement to make given that the purpose of taking the AP exam is to bypass Calculus 1 in college, but according to the MAA's findings, nearly 75 percent of the students who take the AP AB exam and attend a four-year college score 3 or less (out of 5), and almost 60 percent of the students who take the BC exam score 2 or less [3]. Because the College Board calibrates the AP scores so that a score of 3 on the AB exam corresponds to C work (The AB exam is usually one point higher than the BC exam), this means that the majority of students who take the AP exam did, at best, C work in their high school calculus class. The strongest students are testing out of Calculus 1 and starting their college mathematics in a subsequent course, while those who scored low on the AP exam are retaking Calculus 1 in college.

Moreover, because Calculus 1 is foundational to many different majors, it is populated with first-year

students at the beginning of their journey toward a future career. Some of these students are unsure of their major while others may even be questioning whether or not college is right for them. Additionally, many first-year students have not established good study habits. For these reasons, many institutions administer a placement exam to students who wish to enroll in a college calculus class. Students who are deemed not ready for Calculus 1 by a placement exam are often encouraged to enroll in some type of remedial program. While the MAA's report on college calculus identifies different types of placement strategies and even offers advice on what a successful placement strategy looks like, it does not provide any data on the percentage of students who are deemed not ready for calculus by some type of placement strategy.

During my tenure at Messiah College, the Mathematics Department has implemented several different types of placement strategies. Each strategy included a placement exam that was written in-house. This exam has remained virtually unchanged for the past 15 years, though we made the switch from pencil and paper to administering the exam on-line in 2013, and this past year we changed the name of the exam from placement exam to proficiency exam (for consistency, I will continue to refer to our exam as a placement exam). Using data that was collected since moving to the on-line exam, that is between 2013 and 2017, we found that almost 30 percent of the students (n = 342) in our Calculus 1 course scored less than 60 percent on the placement exam (this does not include students who withdrew from the course). Moreover, a statistical analysis of the data suggests that our placement exam has done well at identifying those students who are going to struggle in our Calculus 1 course.

Although our placement exam has done well at identifying those students who are likely to need supplemental instruction in order to succeed at calculus, our strategy for delivering that instruction has not been as effective. When I first arrived at Messiah College in 1993, there wasn't even a placement exam, much less a course in which to place students who were not ready for college calculus. Students were placed into Calculus 1, even if their algebra and trigonometry skills were not calculus-ready. Not surprisingly, these students struggled in Calculus 1. An internal assessment of our calculus sequence led to several changes in our Calculus 1 course, including the creation of a common final exam, a placement exam, and an option for students who were not ready for calculus. At first, students who were deemed not ready for calculus were placed in a traditional precalculus course. Our second attempt placed these students in a two-semester stretch-calculus course that included precalculus remediation delivered in conjunction with the standard material in a Calculus 1 course. Our current approach, which we have been using for six years, allows these students to remain in Calculus 1, but requires an additional one-credit course that emphasizes algebra and trigonometry in the context of the problems encountered in the calculus homework.

While some students have benefited from each of the strategies mentioned above, none of these strategies has been noticeably successful as a whole. Unfortunately, we have no data to substantiate this claim for the first two strategies, but the fact that each was abandoned to pursue a new strategy is evidence of at least the perceived ineffectiveness of both strategies. Although we (Messiah College) have no data to substantiate our claim about the ineffectiveness of our precalculus course, the MAA's report cites evidence that precalculus as a remedial college mathematics course has not been very effective in getting students on track to take Calculus 1. The report notes that there is ample documentation showing that "precalculus as currently taught in most post-secondary institutions in the United States does very little to improve student chances of success in Calculus 1 and can actually be detrimental" [3]. The report also mentions another problem with the traditional precalculus course, one that we found to be true at Messiah College as well: many students who intend to study calculus and who do well in precalculus, do not go on to take Calculus 1 after successfully completing their precalculus course. The report cites

multiple studies with attrition rates as high as 65 percent for students who did well in precalculus. The strategy that is currently in place at Messiah College has not fared much better. Analysis of data that was collected at Messiah College between 2013 and 2017 showed no statistically significant evidence that our current approach increased the probability of success in our Calculus 1 course.

With this context in mind, I would now like to begin to answer the question of how students can find meaning in their study of calculus. I will argue that success (earning a good grade) is not enough to find meaning and will suggest a strategy that will help every student, even those who struggle with the material, find meaning in their study of calculus.

3 Finding Meaning in Calculus

Much good work has been done and is being done to help students succeed in Calculus 1. Personally, I have been involved in this work for years. I have written placement exams, researched and implemented three different remedial strategies, made use of technology to aid in student learning, and used some of the latest research to inform my teaching. My latest writing project is a calculus pretext that uses simple examples and simple language to expose students to course content before they see it in lecture. This pretext is not intended to replace the traditional calculus text but to supplement it. After using the pretext for a semester, I surveyed my students to see if they found the text useful. The results of the survey showed that some of my students found the pretext helpful, but others did not, and a good number of my students did not even make use of this resource. I have found this to be true in general, regardless of the resource. Whether it is the calculus pretext mentioned above, the slides I created to summarize my lectures and give students a visual representation of the problems presented in class, or the reference sheet I created to help students struggling with trigonometry, I put a lot of effort into creating the resource, and only some of my students use it and find it useful. Why is this? That is, why are some students motivated to use these resources and others are not?

The authors of *How Learning Works* note that "students' motivation generates, directs, and sustains what they do to learn". The authors add, "the importance of motivation, in the context of learning, cannot be overstated" [1], especially for college students who have a new found freedom to determine what, when, where, and how they study and learn. How can a lack of motivation be overcome in those students who are not inspired by the thought of studying calculus? According to the authors of *How Learning Works*, when a goal is set before a student, the subjective value that student associates with achieving that goal is a key factor in motivating that student to pursue that goal. The higher the value a student places on a particular goal, the more motivated that student will be to pursue that goal when confronted with multiple goals [1]. At this point in my career, I am not interested in creating another resource that is likely to help only some of my students. I am beginning to think a bit more holistically about some of the problems facing students taking a college calculus course. Rather than focusing on what I can do to fill a gap in student understanding by creating another resource, I want to focus on what the student can do by examining his or her value system in light of a Christian view of learning.

I am not alone in this endeavor. The importance of helping students examine what they value as part of their education was one of the main themes of the June 2015 issue of *Perspectives on Science and Christian Faith*. This particular issue of the journal was dedicated to mathematics, and Russell Howell was invited to write the lead article. Howell's article, *The Matter of Mathematics*, made use of Arthur

Holmes' [6] four categories of faith integration [1] to stimulate conversation regarding mathematics and faith. James Peterson, editor of the journal, noted that Howell had much to say about the interaction of faith and mathematics, especially at the metalevel. I found this to be true, with many of the questions posed by Howell having a philosophical flavor to them. But Howell also posed several questions related to teaching, noting that "additional Christian perspectives are needed in evaluating the ever-increasing approaches to education" [7]. Despite Howell's emphasis on ideas with a philosophical theme, three of the four responses were related to teaching, with two of those responses offering a Christian perspective on pedagogy.

The authors of these two responses, Valorie Zonnefeld and Joshua Wilkerson, both address the idea of subjective value by considering in some way the age-old question students have asked about mathematics: "When will I ever have to use this?" In her response, Zonnefeld says that it is imperative that Christian educators can answer this question because "some students do not readily see the beauty in mathematics, but they may be drawn to its incredible utility" [11]. This statement implies that value can be found in some aspect of mathematics itself. That is, some students may find value (and hence motivation) in the beauty of mathematics while others find value (and hence motivation) in the utility of mathematics.

As an applied mathematician, I have always valued the utility of mathematics. I find great satisfaction in creating a mathematical model that does well at describing reality. My work building a mathematical model of the foot and shoe for Nike Inc. is among the most satisfying of my career. For this reason, I have often looked to the utility of mathematics to inspire my students. On the other hand, I have found the beauty of mathematics to be less inspiring, though as I have matured as a mathematician, this aspect of mathematics has becoming increasingly more attractive. My point is that value is not only a function of the individual but also of that individual's experience. In other words, what a person values may change as a person matures. While some students may find value in certain aspects of mathematics itself (i.e., its beauty or its utility), I do not believe that most of my Calculus 1 students are at a point in life where they assign value to their study of mathematics in this way.

Therefore, if I want to motivate my students, I must first identify and understand what they value. But it is not enough to understand what my students value if their values are misplaced. In this case, I must help them establish an appropriate value system. The authors of *How Learning Works* suggest six strategies to establish value [1].

1. Connect the material to students' interests.
2. Provide authentic, real-world tasks.
3. Show relevance to students' current academic lives.
4. Demonstrate the relevance of higher-level skills to students' future professional lives.
5. Identify and reward what you (the teacher) value.
6. Show your own passion and enthusiasm for the discipline.

I agree with every strategy in this list. I try to get students excited about the material they are learning by showing excitement. As stated earlier, I find enjoyment in constructing models of the real world

[1] The four categories proposed by Holmes are the attitudinal, the ethical, the foundational, and the worldview.

and seek to bring those models into the classroom. I understand the importance of making the material relevant to students' interests, current academic lives, and future careers. Each of these strategies can help students find value in their study of calculus. However, I think there is potential danger here as well. Care needs to be taken to ensure that students do not view their education with only self-serving eyes. In her response to Howell, Zonnefeld hints at this danger. She suggests that when too much emphasis is placed on the student in the education process, it can feed the distorted view of individualism that exists in western culture, a view that finds value only in things that are self-serving. She argues for a "subject-centered" approach to learning, where "God's truth takes center stage" [11].

Personally, I like the subject-centered approach described by Zonnefeld. However, I believe she is idealistic in its implementation. At one point she says, "Curiosity along with cognitive dissonance are harnessed to draw students in to learn more about topics in mathematics" [11]. She then gives an example, asking if it is possible to add four odd integers and obtain a sum of 19. Her description kindled my curiosity and drew me in, but I found myself wondering if it would do the same for my calculus students at 8:00 in the morning.

Wilkerson is not as idealistic in his response to Howell. He believes that most students who ask "When will I ever use this?" have already formulated the answer in their minds. That answer goes something like this: "I will never use this, so learning it is a waste of time". He then suggests that when students ask this question, what they really mean is, "Why should I value this?" [10] I tend to agree with Wilkerson. That is, I believe many of the students in my Calculus 1 class want to know why they should value calculus. They don't simply want to find value in calculus they want to know why they should value it in the first place.

Having said this, I believe that most of my students have already assigned some sort of value to calculus anyway. I believe this is true because they are there; that is, they were motivated to enroll in Calculus 1. To be sure, that value may be misplaced. For example, some students assign value to calculus because it helps them earn the degree they want to earn, while others value calculus because of the money that they will earn in a particular profession, and still others value calculus because it earns their parents' approval. In the past, I have looked to the utility and beauty of mathematics to help students find meaning in their study of calculus. Though this may still be a worthy goal, it will only motivate those students who value the utility and beauty of mathematics. This is not true for many of my Calculus 1 students. I now realize that before I can help these students find meaning in their study of calculus, I must first help them understand why they (as Christians) should value calculus in the first place.

Helping students understand why they should value calculus will enable them to better assign value to their efforts to learn calculus. This is crucial because misplaced value usually leads to lack of meaning. The writer of Ecclesiastes found this to be true of his work: "Yet when I surveyed all that my hands had done and what I had toiled to achieve, everything was meaningless, a chasing after the wind; nothing was gained under the sun" (Ecclesiastes 2:12). A peek back through chapter two of Ecclesiastes sheds light on what the writer valued and hence his state of mind. He valued pleasure (verse 1), achievement (verses 4-6), possessions and wealth (verses 7-8), and status (verse 9). Because these are some of the same values that motivate many students' pursuit of an education, the book of Ecclesiastes is a great place to start a conversation with students about values.

And that is what I plan to do. In other words, I believe that if we are to see progress toward making calculus a pump and not a filter, it has to begin by correcting the flawed value systems that motivate many of our students to study calculus in the first place. The answer is not another resource or innovative

approach to teaching calculus (though these are good things), but Christian educators willing to help students learn the subject matter in the context of biblical values. In the *Abolition of Man*, C. S. Lewis writes, "education without values, as useful as it is, tends to make man a more clever devil" [8]. It is foolish to assume that students' value systems are biblically intact when they are constantly confronted with the value systems of the world. In his book, *Faith and Learning on the Edge*, David Claerbaut argues that "the ability to apply Christian values and analysis to what one studies and learns" [4] is of utmost importance in every discipline, including mathematics. I know of no better tool than Scripture itself to help students (and me as well) see the flaws in what we value. Hebrews 4:12 says, "For the word of God is living and active. Sharper than any double-edged sword, it penetrates even to dividing soul and spirit, joints and marrow; it judges the thoughts and attitudes of the heart".

In order to help students work through an examination of their value systems, I have written twelve reflection exercises consisting of questions, personal thoughts, and Scripture emphasizing learning from a Christian perspective. One such exercise uses the Ecclesiastes passage mentioned above to help students take an honest look at why they enrolled in a calculus course in the first place. A second exercise focuses on a proper foundation for an education. Proverbs 9:10 says that the fear of the Lord is the beginning of wisdom. In this exercise students are asked to consider the foundation of their education and what it means to make the fear of the Lord the beginning of their pursuit of wisdom. A third exercise focuses on distraction. Hebrews 12:1 tells us to fix our eyes on Jesus, the author and perfecter of our faith. In this exercise, students will be asked to consider those things that tend to distract them from pursuing wisdom, with an emphasis on technology (e.g., phones, video games, and social media).

4 Personal Observations

I conclude this article with an example of one of the reflection exercises that I intend to use with my Calculus 1 students and a brief summary of my experience to this point trying to implement these ideas in the classroom. The goal of the example exercise that follows is to have students examine their assumptions about learning mathematics and contemplate how those assumptions might impact their learning. Students are asked to complete the following:

1. Write down everything that you assume to be true about this class.
2. What does it take to be successful in a mathematics course?
3. In her book, *Mindset: The New Psychology of Success*, Carol Dweck distinguishes between a fixed mindset and a growth mindset [5]. What is your mindset with regard to mathematical ability? To answer this question, identify which of the following statement(s) best describe your understanding of mathematical ability:
    - It is something very basic about me that I cannot change very much.
    - I can always substantially change my ability to do mathematics.
    - Someone can learn new things, but they can't really change their ability to do mathematics.
    - No matter how much mathematical ability someone has, they can always change it quite a bit.
4. Read the parable of the talents found in Matthew 25:14-30. What assumptions did the man who received one talent make? How did his assumptions impact his actions?

5. In his talk identifying characteristics of a successful education system, Andreas Schleicher summarizes two different responses (see below) that he received to the question: What makes you successful at mathematics [9]?

   - One response suggested that success is all about talent and that if someone was not a genius at mathematics, that person should study something else.
   - A second response suggested that success is a direct result of study and trust in the instructor.

   Schleicher attributes one of the responses to American students and the other to Asian students. Which response is the response of the American students? Why do you think the response of American students is different than the response of Asian students?

I have not yet had a chance to use this particular reflection exercise with my Calculus 1 students, but I have been able to begin implementing my plan to help students examine their value systems. This past semester, I piloted the ideas mentioned in this article in a small group consisting of junior and senior engineering students. We met weekly (outside of the classroom) throughout the semester. I distributed the exercises to the students prior to our meeting and during our weekly meetings we discussed responses to the questions posed in the exercises. Three of the five students involved regularly engaged in the discussion. Two students offered little input to the discussion, but all agreed that the exercises were helpful.

In addition, I focused my devotional time in Calculus 1 (the first five minutes of class) on the same questions found in the exercises mentioned above. However, unlike the small group mentioned above, I did not distribute the questions to the students beforehand, and I did not allow for in-class discussion. Despite these circumstances, I did see some interesting results. One of my students had this to say about the class: "I would like to formally thank you for the way in which you taught [Calculus 1] this semester. I truly appreciate your passion for mathematics and your desire for the success of each and every one of your students. There were multiple times when I wanted to give up because Calculus was no longer necessary for my new field of study, however you inspired me to keep working. Thank you for the short messages you began each class with. Although some people may have looked past these, I found great worth in each bit of wisdom you shared with us. Please continue to teach the way you are now. Many will benefit from the way in which you are using the talents with which God has gifted you." Another student asked if I would hold him accountable in his struggle with pornography, a first in all of my years of teaching.

To summarize, I am encouraged by the results I saw this past semester. I am not teaching Calculus 1 this semester, but I plan to use these exercises in a first-year seminar for mathematics majors. I am hoping the experience in a seminar setting will allow me to not only answer some of the questions I have about implementing my plan in a large Calculus 1 class, but it will also allow me to refine some of the questions in the exercises.

References

[1] S.A. Ambrose, M.W. Bridges, M. DiPietro, M.C. Lovett, M.K. Norman, and R.E. Mayer. *How Learning Works: Seven Research-Based Principles for Smart Teaching*. Jossey-Bass higher and adult education series. Wiley, 2010.

[2] David M. Bressoud. Ap calculus: What we know. *2009 The Mathematics Association of America*, 2009.

[3] David M. Bressoud, Vilma Mesa, and Chris Larson Rasmussen. *Insights and Recommendations from the MAA National Study of College Calculus*. MAA Press, 2015.

[4] David Claerbaut. *Faith and Learning on the Edge*. Zondervan, 2004.

[5] Carol S. Dweck. *Mindset: The New Psychology of Success*. Random House Incorporated, 2006.

[6] A.F. Holmes. *The Idea of a Christian College*. Eerdmans, 1987.

[7] Russell W. Howell. The matter of mathematics. *Perspectives on Science and Christian Faith*, 67(2):74–88, 2015.

[8] Clive Staples Lewis. *The Abolition of Man*. Zondervan, 2001.

[9] Andreas Schleicher. What are the keys to a successful education system? http://www.npr.org/2017/08/11/541644277/andreas-schleicher-what-s-the-secret-to-a-successful-education-system.

[10] Joshua B. Wilkerson. Cultivating mathematical affections: The influence of christian faith on mathematics pedagogy. *Perspectives on Science and Christian Faith*, 67(2):111–123, 2015.

[11] Valorie Zonnefeld. Practical applications of an integrally christian approach to teaching mathematics. *Perspectives on Science and Christian Faith*, 67(2):124–135, 2015.