Faith Integration Projects for First-year Students

Abstract: Faith Integration Projects for first-year Students (Doug Phillippy, Messiah College)

This talk will consider the use of projects to motivate students to think deeply about how their faith connects with mathematics. This talk will begin by describing what a faith integration project is, including the goals and objectives of such a project. The talk will briefly describe a number of projects written by the speaker, with a more detailed look at one of those projects. The talk will conclude by discussing how these projects are being used to assess how students are doing at articulating a maturing understanding of the connection between faith and mathematics.

This paper will describe projects that are intended to motivate first-year students to think deeply about how their faith connects with mathematics. It is important to understand that these projects form one section of a 350-page text entitled, *The study of Mathematics: Developing a mature understanding of mathematical thought, with consideration of Christian faith and vocation*. This text, which has not yet been published, has an intended audience of first-year mathematics majors. As such the projects described in this paper are just one component of a larger work that is intended to be a tool to help first-year mathematics majors begin their study of mathematics from a Christian perspective.

This text, which is now in its third draft, has been a nine-year project which is nearing its completion. In fact, I am currently seeking a publisher for this text. There are three main themes woven throughout the text. Obviously, one of those themes is mathematics. I have included topics in the text that I believe will be of interest to first-year mathematics majors. Some of these topics are foundational to the study of mathematics, and others, though they are not typically found in the curriculum of a first-year student, were chosen because they are thought-provoking. The second major theme interwoven throughout the text is that of the Christian perspective. We will consider the study of mathematics in light of the Christian faith, asking how faith influences the study of mathematics and in turn how the study of mathematics can impact faith. The third theme is that of my life story. Throughout the text, I share my journey as a Christian mathematician, noting some of my struggles and highlighting what I have learned along the way.

The main body of the text is divided into four sections. Chapters 1 and 2 form the introduction to the text. The second section (chapters 3-9) of the text focuses on applied mathematics covering topics such as check digits, graph theory, number theory, and problem-solving. The third section (chapters 10-13) of the text focuses on theoretical mathematics covering topics such as logic, proof and small axiom systems. The fourth and final section of the text (chapters 14-21) consists of what I will call faith integration projects. These projects cover a variety of mathematical topics and are designed to promote discussion in the area of faith integration. They are the focus of this paper.

Most of this paper consists of one example of a faith integration project that is included in my text. It will also include a brief description of several of the other projects. But before we consider these projects, I need to make a few introductory comments regarding the nature of these projects. I will do so in light of several key characteristics of faith integration mentioned throughout the text of which these projects are a part. My intent is not to develop these characteristics fully here. Still, I mention them here because they provide the framework for understanding the nature of these projects. These characteristics include the two-way nature of faith integration, the importance of community in faith integration, and faith integration as a scholarly project. These characteristics form the foundation on which the projects are built.

One characteristic of faith integration that is emphasized in chapter 12 of the text is the two-way nature of faith integration. By this I mean that integrating faith with the study and practice of mathematics should include both the insight that a Christian worldview brings to the discipline of mathematics and the contributions of the discipline to a Christian view of reality. In the text, I suggest that of these two possibilities, it is often easier, especially at the undergraduate level, to consider the impact that the study of mathematics has on a Christian view of reality rather than vice versa. I do not want to develop the reasons for this here, but I mention it because the majority of the projects that I have written focus on the impact that mathematics has on faith. In this paper, I will consider one such project, a project that suggests our study of mathematics can help to develop our intuition with regard to the infinite, including our eternal God.

A second characteristic of faith integration is that it is most effective when it takes place in community. In my text I suggest that faith integration is a process, noting that initial attempts at faith integration may result in less than noteworthy results. Still, any attempt at faith integration, even a flawed attempt, can prove beneficial, if for no other reason than to promote discussion among those who actively seek to connect faith and learning. Not only does this allow the architect of the attempt to practice faith integration, but it draws from the knowledge of an entire community to sharpen that attempt.

The projects presented in my text attempt to model this characteristic of faith integration. Each project is meant to encourage dialogue. To accomplish this, I begin the conversation with some of my own thoughts on a particular topic. These thoughts are intended only to initiate the dialogue, not to provide the reader with an expert's final analysis of the topic. In particular, each project consists of a short essay that is an attempt on my part to relate faith and mathematics in some way. These essays discuss a variety of mathematical topics appropriate for undergraduate students. For the most part each essay is self-contained and no successive progression through the material is required.

Nevertheless, because the essays are designed to promote discussion and not provide an answer, my hope is that they will provide a basis for further work in the area of faith integration. So, the essay is only part of the project. Each project has the potential for reader participation. Each project will begin with a question and include some of my thoughts as to how that question might be answered. As such, my essays provide an opinion and not "the answer" to the question.

The key to these projects really is the reader's response. My role is only to begin the conversation. The reader's response may be a critique of the answer that I have provided in the essay or it may be the reader's own answer to the question, or it may be both. It may even be the reader's initial thoughts to some other question that the essay prompted her to consider. In any case, the goal of each essay is to engage the reader in connecting faith and mathematics.

While the primary goal of the essay portion of each project is to begin a conversation with the reader regarding faith and mathematics, my writing serves an additional purpose. In particular, the essays in the text are given to serve as a pattern of the type of work that is expected to enter into the dialogue. In the text, I emphasize that faith integration is **any** attempt by an educator or student to relate one or more of the academic disciplines (not necessarily the individual's major or specialty) to a biblical worldview. I argue that faith integration should not be limited to a narrow range of approaches. I note that this is especially true for those who are in the initial stages of thinking about faith integration. Still, I suggest some guidelines for work to be done by a student in response to the essay portion of the project.

My goal is to move the student toward William Hasker's definition of faith-learning integration. This definition describes faith integration as "a **scholarly** project whose goal is to ascertain and to develop integral relationships which exist between the Christian faith and human knowledge, particularly as expressed in the various academic disciplines". So while I place very few restrictions on my students in other settings with regard to faith integration, these projects raise the bar a bit. This is not to say that the reader's response need be as extensive as the initial essay itself, nor is the goal to produce some paper ready for publication. Instead, the goal is to think seriously about faith and mathematics, connecting the two in some fundamental way.

I offer some guidelines as to what it means to produce a project that is scholarly in nature. At a minimum, the dialogue should be a response to some of the work already done in the text. It might seek to answer one of the questions asked at the conclusion of an essay, or it may be a response to the essay itself. At a more serious level, the dialogue might be original work, not a follow-up to discussion in the text. As such the projects in the text do not provide the material for the work being done but serve as a guideline for the type of work that might be done. For example, the reader may choose to use the project presented at the end of this paper entitled "the Infinite and Intuition" to construct a multi-step exercise that promotes active learning, with the reader discovering a faith-learning principle as he or she works out the exercise.

Ultimately, the purpose of this section is to help the reader think deeply about mathematics and faith, whether by responding to the author's thoughts or by producing original work. In either case, the discussion should include appropriate worked-out mathematical examples as well as an overview of the topic being considered, including pertinent definitions and theorems. Discussion should include references to Scripture and appropriate faith-related definitions. It might seek to identify which of the faith integration approaches described in the text best fits the approach expressed in the dialogue. It also might include what others have written and said about the topic. A student project need not include all of the above elements, but it should consist of those that are necessary to make the dialogue appropriate for an academic discussion.

I will now turn to the projects. Currently my text includes 8 such projects. Before considering one of those projects in detail, I will give a short description of several of those projects.

Infinity and Time - This project asks the question "What role (if any) should a Christian perspective have in discussing the solution to a problem that requires time to approach infinity?" Several Christian perspectives of time are discussed including the classic Augustinian view that time is finite. The author argues that this viewpoint of time impacts the way Christians should think about infinite limits. Moreover, he argues that such a discussion has a place in the mathematics classroom. Though the focus of this project is on the nature of time and the concept of an infinite limit, the underlying theme is how Christian thought can be introduced into the mathematics classroom.

The next three projects discuss the role that the infinite plays in mathematical reasoning and suggest that by studying the infinite in mathematics we can gain insight into the Christian faith. These three projects are related through this common theme and are best understood if considered as a whole.

Overcoming Paradox - In this project, our emphasis will be on paradox and the role the infinite plays in both overcoming and creating paradox. We begin by exploring what can happen if the infinite is excluded from the reasoning process. We suggest that excluding the infinite from the reasoning process or even having an improper view of the infinite can lead to paradox. In particular, we discuss some of the paradoxes that came about from the Greek understanding of time and space. The Greek understanding of time and space was limited by the way the Greeks viewed the very small. We use the geometric series to expose some of their misconceptions. In so doing, we illustrate how a proper inclusion of the infinite in the reasoning process can overcome paradox, thus giving a better understanding of reality.

The Infinite and Intuition - In this project we ask how well our intuition does at answering questions related to the infinite. We explore the possibility that the study of the infinite in mathematics can develop our intuition with regard to God and things eternal. We note that the result of an infinite process can be counter-intuitive and sometimes even paradoxical. In this project, we develop the integral test and consider several problems that test our intuition and challenge our understanding of reality. This project is included in its entirety at the end of this paper.

Taming the Infinite - Because of the sometimes counter-intuitive nature of the infinite, we suggest in this project that careful consideration must be given to underlying assumptions when working with the infinite. In particular, we consider the alternating series and the conditions necessary to guarantee the uniqueness of a sum. Using this as a backdrop, we suggest that care must be taken when studying the infinite not to trivialize it. We suggest that a blind application of laws that hold in a finite realm to God can lead to a trivialization of God.

Higher Dimensions and Paradox – In this project we ask if the study of dimension in mathematics can help to resolve paradox within the Christian faith. In particular, we ask how the study of higher dimensions in mathematics might shape our view on the ideas of predestination and free-will that are important to the Christian faith. A sphere's attempt to convince a square that there is a third dimension (taken from Edwin A. Abbott's book *Flatland*) serves as the impetus for seeking an answer to these questions. In particular, the inability of the square to understand three-dimensional space is used to motivate how time and space-bound humanity might find it difficult to understand a God that is not bounded by space and time.

Numbers and the Bible – In this project we ask what role the idea of number plays in our faith. In particular, we ask if the study of number in Scripture is appropriate. We note a couple of claims about number in Scripture including the claim that Scripture is written in a numerically significant way, the claim that certain numbers have more than just quantitative meaning, and the claim that the use of number in Scripture is reason to believe that Scripture contains error.

I conclude this paper with one of my projects as it appears in my text.

The Infinite and Intuition

He has set eternity in the hearts of men; yet they cannot fathom what God has done from beginning to end. Ecclesiastes 3:11

Question: Can the study of mathematics help a Christian develop intuition with regard to understanding God and eternity?

In the project entitled "Overcoming Paradox", we noted that exclusion of the infinite in the reasoning process can create paradox. We saw that the exclusion of the infinite in Greek mathematics led to paradox and a flawed world view. Moreover, we also argued that in the absence of the infinite, many statements in the Bible lose much of their meaning or they become paradoxical. In this chapter we suggest that care must be taken when incorporating the infinite into the reasoning process. Infinite processes can yield results that are quite surprising and sometimes even paradoxical. In this chapter, we examine several infinite processes that will test our intuition and challenge our understanding of reality.

However, before we consider these examples, I need to define what I mean by intuition. Davis and Hersh note that the word intuition is used by mathematicians in many different ways. In their book, *The Mathematical Experience*, they list at least 6 different ways that this word can be used, including as a substitute for rigorous proof, a brilliant flash of insight, visual, relying on a physical model, and incomplete. *The American Heritage Dictionary* also lists several meanings of "intuition" including, "the act or faculty of knowing without the use of rational process" and "sharp insight". By intuition, I simply mean "insight into". I am not too concerned with how this insight may come about. It may be some mysterious insight that few others seem to possess or it may be an insight that has been developed through use.

When it comes to intuition about God, Scripture suggests that we as natural human beings at best have a flawed intuition. The prophet Isaiah recorded the following statement of God regarding the thoughts of God and humans, "'For my thoughts are not your thoughts, neither are your ways my ways,' declares the LORD. 'As the heavens are higher than the earth, so are my ways higher than your ways and my thoughts than your thoughts.'" God can be described as omniscient, omnipresent, omnipotent, and eternal. These attributes are closely related to his infinite nature and make him difficult to comprehend. Answers to questions regarding his triune nature are not easily formulated. Still, people have sought them out. Theologians search the Bible and interpret the Scriptures to gain an understanding of the eternal. Then they attempt to describe the God who existed before the world began (John 17:5), and is not bound by space (I Kings 8:27). Is it any wonder that their descriptions are incomplete? As Dorothy and Gabriel Fackre have noted, human beings are often limited or "tripped up" by the language of their experience. That is, the language, experience, and knowledge of finite beings are often inadequate in describing an infinite and eternal God. Nevertheless, in an attempt to understand God and His creation, many authors have written books on Christian theology trying to answer these and other questions.

Likewise, mathematicians have long sought to gain an understanding of the infinite. In fact, mathematics has been called the science of the infinite. Mathematicians construct axiomatic systems and use symbols and operations within the framework of those systems to gain an understanding of the infinite. They use the infinite in their reasoning and routinely perform processes of infinite length. As a result, these same mathematicians sometimes stumble over paradoxes that arise within their carefully constructed axiomatic systems. The uncertainty that these paradoxes raise has ramifications that reach to the very foundations of mathematics. Nevertheless, in an attempt to describe the world around them, mathematicians continue to produce work that has its basis in those foundations.

What happens when a process is repeated an infinite number of times? This question is itself of a paradoxical type, since we cannot answer it by doing what it asks, regardless of what the actual process is that is to be repeated. However, as we have already seen in our discussion of the geometric series, the result of an infinite process can be described quite precisely. As another example, consider the derivative. A study of calculus reveals that if the value of Δx moves infinitely close to 0, then the value of $\frac{f(x+\Delta x)-f(x)}{\Delta x}$ approaches $f'(x)$ (the derivative) assuming that $f(x)$ is a differentiable function of x. In calculus, we study infinite processes such as summing an infinite number of terms or moving infinitely close to a point by using the concept of the limit. In other areas of mathematics, infinite processes are studied with different tools, such as the idea of a random variable distribution in probability.

The subject of probability is a good place to begin our study of intuition. Imagine, for instance, that two young children, Ben and Amy, are flipping a coin. Ben wins when heads comes up and Amy wins with tails. If the first flip comes up heads, both kids will consider that a natural event in the game. If the first 5 flips are heads, Ben will be excited and Amy dismayed. If the next five flips are also heads, Amy will start to get suspicious. And if the *next* five flips are still heads, even Ben will know that the coin is probably not fair. What underlies their suspicion is an intuitive understanding that a fair coin should land heads up in close to half of all flips, over the long run. Mathematically, we say that the proportion of heads should approach ½ as the number of trials approaches infinity. This is an example of the Law of Large Numbers at work in probability theory. It is a tool for considering an infinite process that involves randomness, unlike most problems in calculus.

Now, suppose I perform an experiment in which I flip a coin 5 times and ask the reader to predict the results. There are 32 possible outcomes to this experiment, each equally likely to occur. They are listed below:

HHHHH, TTTTT
HTTTTT, THTTT, TTHTT, TTTHT, TTTTH
THHHH, HTHHH, HHTHH, HHHTH, HHHHT
TTHHH, THTHH, THHTH, THHHT,HTTHH, HTHTH, HTHHT, HHTTH, HHTHT, HHHTT
HHTTT, HTHTT, HTTHT, HTTTH, THHTT, THTHT, THTTH,TTHHT, TTHTH, TTTHH

Although each of the above events is equally likely to occur, intuition may make some of the outcomes more likely to be chosen by the reader as a prediction. Let me explain. Our intuition tells us that flipping a fair coin is not very likely to result in five heads. This will happen only 1in 32 times. Our intuition also tells us, that though it is more likely to obtain four heads than five, it is also more likely to obtain three heads than four. Because of this, and our intuition that the flipping of a coin should produce random results, the events displaying “more randomness” may be more likely to be chosen as predictions by the reader than the events consisting of less random patterns. In other words, even though the events described by HHHHH and HTTHT are equally likely to occur, the appearance of randomness in the latter event makes it more likely to be chosen as a predicted result of flipping a fair coin five times.

The above examples illustrate that our intuition can be beneficial in understanding a problem, but it also can be misleading. The following exercise is meant to test your intuition.

Exercise: How Many Threes?

What percentage of whole numbers have at least one 3 in their base 10 representation (for example 127 does not have a 3 in its base ten representation and 333 does)?

Your Guess:____________

a. In an attempt to help answer this question, complete the following table (I have completed the first two rows; no entry is needed in the shaded cells). Complete each row before moving to the next row. Work from left to right across each row. Each cell should contain the number of whole numbers with a three in the appropriate digit. Do not count a number twice. For example when considering all the two-digit numbers (row 2 below), the whole number 33 should be counted in the 1st (leading) digit column and not again in the second column.

b. Try to identify a pattern in each column. In particular answer the following questions:

What happens to each entry as we move down the column to the next row? In other words what is the relationship between the entries in a given column?

What is the pattern for the rightmost entry in each row? In other words, how do the numbers increase along the diagonal adjacent to the shaded cells?

		Number of 3's in the: (from left To right)						
	Number of whole numbers in this range	1st digit	2nd digit	3rd digit	4th digit	5th digit	Total number of whole numbers with at least one 3 (sum of columns 3-7)	Percentage of whole numbers with at least one 3
All 1 digit numbers (0-9)	10	1					1	.1
All 2 digit numbers (0-99)	100	10	9				19	.19
All 3 digit numbers (0-999)	1000							
All 4 digit numbers (0-9999)	10000							
All 5 digit numbers (0-99999)	100000							

0	10	20	30	40	50	60	70	80	90
1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	95
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99

c. Consider the last row in the table. Write the sum of 5-digit whole numbers with at least one of their digits being a "3" by making use of the patterns established in part b).

d. Generalize your result in part c) to account for a number that is *n* digits long.

e. Let $p(n)$ represent the percentage of whole numbers with at least one digit being a 3. Use technology and blot $p(n)$ for n ranging from 1 to 50.

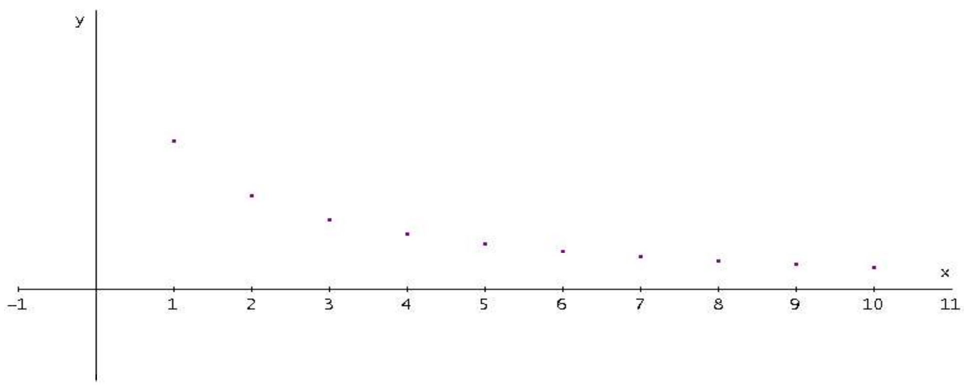
f. Let n approach infinity and make use of the sum of a geometric series to calculate the percentage of whole numbers with at least one 3 in their base 10 representation.

How did your intuition match up with reality in this exercise? If you are like most people, probably not too well. Perhaps this is because intuition is influenced by the experiences of everyday life. Most people don't experience numbers that are very large in magnitude on a regular basis. Therefore it is reasonable to expect that your guess was likely formulated with numbers that are relatively small in magnitude in mind.

We continue to test your intuition in this section by considering several more exercises that have surprising results. But before we turn to those exercises, we need to develop the integral test for infinite series. This is done in the next exercise.

Exercise: Discovering the Integral Test for Infinite Series

Consider the following graph of the first 10 terms (all positive) of an infinite series: $\sum_{n=1}^{\infty} a_n$.



a. Label each point with its x and y-coordinates (k, a_k).

b. Connect the points with a continuous decreasing function $f(x)$ that approaches the x-axis as x approaches infinity.

c. From each point construct a rectangle by drawing a horizontal line of length one to the right of each point. This line is the top of the rectangle, and the corresponding portion of the x-axis is the bottom of the rectangle. What is the area of the rectangle?

d. Write an expression that represents the sum of the areas of the rectangles.

e. Assume $\int_1^{\infty} f(x)dx$ is infinite (diverges), what can you conclude about $\sum_{n=1}^{\infty} a_n$?

f. Assume $\sum_{n=1}^{\infty} a_n$ is finite (converges), what can you conclude about $\int_1^{\infty} f(x)dx$?

g. Explain your reasoning for parts e and f.

By repeating the above exercise with line segments of length one drawn to the left of each of the points in the graph, it can be shown that the converse of each of the statements in parts e) and f) is also true. Thus we arrive at the integral test:

If $f(x)$ is a positive, continuous, and decreasing function for $x \geq 1$ and $a_n = f(n)$ for all n, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)dx$ either both converge or both diverge.

In the following two exercises, we evaluate improper integrals and use the integral test for infinite series to obtain results that defy our intuition and maybe even cause us to question our view of reality.

Exercise: Shopping at the Infinity Toy Store: The p-series

Suppose you purchase a set of blocks at the Infinity Toy Store. This set of blocks is an infinite set of square blocks with dimensions $1x1x1$ inch, $\frac{1}{2}x\frac{1}{2}x\frac{1}{2}$ inch, $\frac{1}{3}x\frac{1}{3}x\frac{1}{3}$ inch, etc. for all positive integer denominators. The question is whether or not this set of blocks will fit in your dorm room. To complete this exercise use your knowledge of infinite series.

a. If the blocks are arranged by stacking them one upon another, beginning with the largest, write a summation that represents the height of the stack.

b. Determine the convergence or divergence of the infinite series found in (a) of this exercise by using the integral test (evaluate the integral). How high is the stack?

c. By completing (b) in this exercise, you should have found that the stack will eventually reach beyond the moon. What does your intuition tell you about the possibility of fitting this set of blocks in your dorm room?

d. Suppose instead of stacking the blocks one on another, you attempt to lay them out on the bottom of your top desk drawer. In this arrangement, no block is to be stacked on top of another. Write a summation that represents the total area of the bases of all the blocks.

e. Determine the convergence or divergence of the infinite series in part (d) by using the integral test. How much area is required in your top desk drawer to store the blocks? To answer this question, sketch a potential arrangement of the blocks.

f. Evaluate $\int_1^\infty \frac{1}{x^p}\,dx$. For what values of p does the integral converge? What can you conclude about $\sum_{n=1}^{\infty}\frac{1}{n^p}$?

Exercise: Another Toy at the Infinity Toy Store: Gabriel's Horn

Suppose you purchase a horn at the Infinity Toy Store that was made by revolving the function $f(x)=\frac{1}{x}$ about the x-axis on the interval $[1,\infty)$. (This object is known as Gabriel's Horn).

a. Show this horn can hold a finite amount of liquid by evaluating an appropriate integral.

b. Set up an integral that represents the amount of paint required to paint the exterior surface of this horn.

c. Explain why $\sqrt{1+\frac{1}{x^4}}>1$, and use this fact to show that there is not enough paint in the universe to cover the outside of this horn.

The exercises in this project were offered to illustrate two principles regarding human intuition as it relates to the infinite. First, because human intuition is grounded in an experience in a finite world, and because that experience is often in the context of quantities that are relatively small, human intuition with respect to the infinite is unlikely to be something that has had opportunity to develop. Second, when it comes to the infinite, some outcomes don't seem to make sense, much less be intuitive. After all, how can cubes that stack higher than the moon fit inside my desk drawer? Or how can an object hold a finite amount of paint and yet be unable to be covered by any amount of paint?

With these exercises in mind, I am ready to answer the question posed at the beginning of this chapter: The study of mathematics can help a Christian develop intuition with regard to understanding God and eternity. The two principles mentioned in the previous paragraph can be applied to my theology. In these exercises, I see anew that my quest to understand an infinite God is hindered by my experience in a finite world. Moreover, my study of the infinite in mathematics enables me to experience the infinite in ways that no other discipline can offer. In other words, my intuition about things that are eternal has a chance to develop. My experience also teaches me to expect the unexpected when it comes to studying God. Outcomes that don't make sense in a finite reality are possible in the realm of the infinite. We will consider some of these outcomes in the next project.

Questions for Further Thought

1. How does the author answer the question, can the study of mathematics help a Christian develop intuition with regard to understanding God and eternity? Do you agree or disagree with his thoughts?

2. Identify one belief that you hold about God which you do not fully understand. In what ways is this belief related to God's infinite nature? Has the discussion in this chapter given you any insight regarding this belief?

3. Read 1 Corinthians 2. Analyze the claims the author makes in this chapter in light of what this passage says about understanding things related to God.

4. Identify one surprising mathematical result that you have encountered which is based in the infinite (not mentioned in this chapter). Does this result give you any insight into spiritual things?

5. Has your intuition ever failed you when it comes to thinking about God? In what ways is God's infinite nature related to this failure?

References

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