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Quantitative Reasoning in the Numbers: A Qualitative Study of Trends in Higher Education Mathematics Curriculum

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QUANTITATIVE REASONING IN THE NUMBERS: A QUALITATIVE STUDY OF TRENDS IN HIGHER EDUCATION MATHEMATICS CURRICULUM

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Abstract

Mathematics occupies a dynamic place in general education curriculum. The subject once served primarily as a language to explain the natural world and more recently adopted more usefulness-centered value in research and society. Changes in K-12 curriculum standards over the last two decades have again reoriented the subject toward quantitative reasoning (QR) acquisition, responding to demands by employers for greater facility with quantitative information and recommendations by researchers with regard to learning acquisition and transfer in mathematics. After defining components of QR, this study examined mathematics curriculum at highly ranked liberal arts colleges and universities across a ten-year period to determine whether higher education institutions have responded to experts' calls and incoming students' shifting preparation. An analysis of course catalog documents revealed positive trends in at least some areas of QR education and raised implications for practice within general education and for future research in mathematics curriculum.

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Chapter 1

Introduction

Give a child a shape sorter and watch as the most intense forms of focus and curiosity emerge. Some of the first skills children learn include recognizing and differentiating shapes, followed quickly by counting and creating patterns. Children build on these foundational skills when they enter school and discover number systems, addition, subtraction, and spatial relationships.

The National Research Council reported that young children “show a remarkable ability to formulate, represent, and solve simple mathematical problems and to reason and explain their mathematical activities,” and they “are positively disposed to do and to understand mathematics when they first encounter it” (Kilpatrick, Swafford, & Findell, 2001, p. 6). Soon, however, too many children lose not only the desire to pursue mathematics but also the formal educational support in their pursuit. In the absence of forming connections between numerical learning and broader understandings of the world and their experiences, students’ sentiments often echo the familiar challenge:
When will I ever use this stuff in real life?

Quantitative Reasoning Defined

The first institutions of higher education in colonial America placed less emphasis on mathematics than on languages and religion, which provided reading and writing skills necessary for careers in education and ministry. However, the young country approached education with a new perspective on scholasticism, and mathematics contributed to the growing pursuit of experimentation, evidence, and practicality (Rudolph, 1990). In the Yale Report of 1828, faculty—under the presidency of a mathematician—vehemently defended the classical liberal approach to education and the cultivation of "all the important mental faculties," balancing the need for science with literature, pure mathematics with art, and "solid learning" with eloquence (Yale College, 1828, p. 7-8). However, by the twentieth century, the German university's influence on research deepened the development of new technologies to ensure prominence in scientific exploration and application. World wars generated the need for military technology, the race to the moon and beyond fueled curiosity, and the Internet and other advancements in communication allowed for instantaneous calculation and application of newfound knowledge. Science, Technology, Engineering, and Mathematics (STEM subjects) soon rose to the forefront of academic pursuit (Herschbach, 2011; National Science Foundation, 2014).

In the past two decades, though, the need for all students to feel comfortable with quantitative information and tasks and their extension into daily life—rather than STEM-specific research and careers—gained ground. Colleges and universities join with other educators to emphasize quantitative literacy as an outcome of higher education. As collegiate educators instruct growing proportions of the population—many of whom have low levels of confidence or proficiency in mathematical abilities when they enter institutions of higher education—they must determine how best to increase their students' level of fluency with numbers.

Terminology in the field of quantitative literacy, however, by no means proves consistent. *Quantitative reasoning* (QR), *quantitative literacy* (QL), and *numeracy* comprise the set of most common terms used. Though the three terms are generally employed synonymously (Vacher, 2014), each one carries a different slant according to its context. Most often, QR refers to the process of problem solving and applying mathematical, numerical, and arithmetic skills to real-life situations (Vacher, 2014). K-12 literature often uses QL on account of how it parallels English literacy and the related movements in education reform. Numeracy often connotes the more mathematical-technical orientation (Wismath & Worrall, 2015) and also appears more commonly in literature from the United Kingdom (UK). The current study used all three of these terms but gave a formal definition under the name *quantitative reasoning*.

Corckroft first defined the term *numeracy* as “the ability to cope confidently with the mathematical demands of adult life” (Wismath & Worrall, 2015, p. 1), expanding on a report published in the UK in 1959, which stated numeracy should mirror literacy (Madison & Steen, 2008). Incorporating reasoning skills and emphasizing the ways in which solid quantitative foundations allow for rational and accurate judgments (Elrod, 2014), QR also enables more forceful communication. Education for QL commonly advocates the use of quantitative skills in fields and courses outside of mathematics (Dumford & Rocconi, 2015; Elrod, 2014; Jordan & Haines, 2005; Orrill, 2001). Orrill (2001) grouped numeracy with reading and writing to create the “triumvirate of competencies that . . . make up the traditional core of literacy” (p. xv), an approach that has implications for mathematics education at every level.

Based on the historical and contemporary conceptions, current research defines quantitative reasoning as *the ability to identify situations outside of a formal mathematics environment in which quantitative skills can contribute to understanding, evaluating, and communicating numerical information and facility in applying practical mathematical skills in those contexts.*

Relevance of Study

In addition to the stress on STEM subjects in research and curriculum, recent studies reveal employers increasingly consider mathematical skills necessary in many job fields. Even as advances in technology turn number-crunching into a computer's position, strong quantitative skills allow employees to analyze and solve problems in the number- and data-rich workplace. The current Geometry-Algebra-Trigonometry-Calculus (GATC) curriculum lacks mastery of reasoning, analysis, and judgment based on numerical information. In response, the movement toward a quantitatively literate populace parallels the emphasis on literacy decades earlier. English literate individuals recognize the shapes of letters and sounds of phonemes but extend their use to convey meaning through language. Similarly, employers and communities seek persons who hold numerical and operational skills but extend those basic units to understand, evaluate, and communicate.

Recent Trends in Education for Quantitative Literacy

Elrod (2014), however, asked,

> What do terms like quantitative reasoning, quantitative literacy, and quantitative fluency really mean for student learning, the curriculum, program development, faculty development, or accreditation? Why is it such an important outcome?

> How do we teach and measure it? Who is responsible for ensuring that students achieve this competency? (para. 6)

Educators at all levels must now attend to these questions, and many have made advancements in defining outcomes and providing resources for implementation. For example, the National Numeracy Network (NNN) formed in 2000 in order to "[promote] education that integrates quantitative skills across all disciplines and at all levels" (The National Numeracy Network, n.d.). *Numeracy*, the biannual journal first published by the NNN in 2008, provides resources and strategies for educators based on research and best practices in QL education.

In 2009, government leaders initiated development of a set of standards for possible implementation across educational district and state lines. The resulting Common Core State Standards (CCSS) included expectations in mathematics that "emphasize coherence at each grade level—making connections across content and between content and mathematical practices in order to promote deeper learning" (American Diploma Project Network, 2012, p. 11). Educators and policymakers hope sound instruction in the CCSS will prepare students better for college and careers.

Purpose and Research Questions

As institutions pursue increased success in QR preparation for students entering the workplace and global community, faculty will become responsible for instruction and mastery in their own classrooms, and institutions will continue to set an expectation and environment conducive to QR development. Institutional leaders will serve as "agents of change in general education requirements" in shaping course and curriculum expectations to adapt to external pressures and "new issues in their environments" (Brint, Proctor, Murphy, Turk-Bicakci, & Hanneman, 2009, p. 611). Madison (2014) noted, “There are no clear guidelines for courses and no generally accepted measures of success” for QR courses (p. 2), so current research continues to develop specific best practices. However, many institutions have already taken steps toward educating for numeracy.

The current study sought to explore the progress colleges and universities have made toward incorporating quantitative reasoning outcomes into the educational experience of students. The following research questions guided the study: To what extent have general education mathematics courses changed over the last decade? Do changes reflect increased attention to quantitative reasoning with regard to analysis of numerical data, application of quantitative skills to problem solving in real-world contexts, and evaluation of arguments and judgments based on quantitative information? In order to provide context for the research questions, Chapter 2 presents a literature review of both historical perspectives on mathematics—indicating the importance of quantitative topics in the development of civilizations and educational systems—and current research within the fields of mathematics and higher education.

Chapter 2

Literature Review

Recent years have witnessed an increased emphasis in higher education on the STEM subjects—Science, Technology, Engineering, and Mathematics—each deemed necessary for an increasingly technology-filled job market. However, all jobs require some level of quantitative competency; thus, employers have called for increased mathematical preparation for potential employees. The Association of American Colleges and Universities (AAC&U) found 55% of employers believed colleges and universities "should place more emphasis on students' ability to work with numbers and understand statistics, and 81 percent believe more emphasis should be placed on the ability to analyze and solve problems" (Berg et al., 2014, para. 1). Employers desire more than computational proficiency. Mathematical reasoning skills, including logic and problem solving in addition to basic arithmetic competence, appear increasingly desirable and fundamental in non-STEM majors. However, the current GATC curriculum lacks mastery of reasoning, analyzing, and judging based on numerical information. In response, the movement toward a quantitatively literate populace rose in ways comparable to the emphasis on literacy decades earlier.

Primary and secondary schools use curriculum reform to address changing needs in society. The recent inception and implementation of Common Core State Standards were designed "to ensure students are prepared for today's entry-level careers, freshmanlevel college courses, and workforce training programs" (Common Core State Standards Initiative [CCSSI], 2015c, para. 2). Schools teach the mathematical skills educators judge necessary for advancement, usually on the typical GATC continuum (Madison, 2004). Over the last fifteen years, institutions of higher education began to follow suit, evaluating their graduation requirements in mathematics, especially for non-mathematics majors. Increased conversation since the turn of the century indicates this topic holds importance not only for mathematics faculty but also for all collegiate educators.

As indicated in Chapter 1, the terms *quantitative literacy*, *quantitative reasoning*, and *numeracy* are generally used interchangeably to describe this competency with quantitative skills and their application in real-world contexts. Most institutions of higher education must now define the place of this new requirement in their curriculum to determine how best to educate their students for demands post-graduation. While the precedent includes quantitative skills in mathematics courses, research supports a more global approach, similar to the inclusion of literacy as an outcome in departments besides English language and literature. As colleges and universities address the changing needs of their students, tools for assessment are being piloted and best practices established. The next several years will determine whether the new vectors in quantitative skills education will support the desired outcomes.

Purpose of Mathematics Throughout History

The place of mathematics in curriculum has not always been relegated to a single school or department. Mathematical ideas existed for millennia as a useful tool in the construction, measurement, and economics of daily experiences. Thales of Miletus, credited as the founder of demonstrative mathematics, revitalized the subject in the seventh and sixth centuries BC, using mathematics to search for "an explanation of the universe" (Sanford, 1930, p. 5). Pythagoras, known and bemoaned by many students as the creator of the theorem bearing his name, established a school of mathematics around 518 BC. Rather than separating the subject from other disciplines, he used mathematics as a language to carry on the philosophical discussions of the age (Suzuki, 2009). His curriculum included arithmetic and geometry as well as music and astronomy (Sanford, 1930), all topics considered in essence mathematical and later comprised the *quadrivium*, or “the way of four” (Nelsen, 2014, p. 105).

Founding the Academy in Athens in 387 BC, even Plato emphasized in his teaching that "mathematics [is] a way to train the mind in deductive thinking" (Suzuki, 2009, p. 27). As a basis for instruction, Plato and his tutee, Aristotle, relied on the *quadrivium*, which became the base of the ancient educational system. Paired with the *trivium*, "the three literate arts of grammar, logic, and rhetoric" (Nelsen, 2014, p. 105), these four disciplines made up the liberal arts, those subjects necessary for mastery by those who wished to become literate citizens.

Contemporary Shifts in Foci of Mathematics

Colleges and universities in recent decades see mathematics through a much narrower lens. Blumenthal (2003) and others raised questions about the role of mathematics in today’s core or general education requirements. The diversification of higher education sequesters mathematics to its own department, often with a basic survey course as the only expectation for non-majors who enter college ill-prepared (Brint et al., 2009) or ill-motivated for technical mathematics study. Students who do enter a calculus or statistics course often believe the "job of a mathematics professor is to prepare them to do the kinds of problems that are going to appear on examinations," (Banchoff, 2002, p. 22), focusing on mathematical content but rarely extending beyond formulas and processes. Proponents of the liberal arts challenge the isolation of the field, noting "mathematics is not some esoteric and technical endeavor" but instead "at some level, a study of how we as human beings think" (Blumenthal, 2003, p. 39).

For centuries, mathematics courses remained part of general education at the college and university level, if only as an homage to "the way it has always been." In 1959, the Crowder Report from the UK first coined the term "numeracy" to describe the "ability to apply quantitative evidence to arguments in broad contexts of personal and public life" (Grawe, 2014, para. 1), placing the topic in public attention. The narrow focus of GATC mathematics widened once more to a Classical emphasis on relation to everyday life and decisions.

Urgent societal needs drive this emphasis, as recent studies find Americans report some of the most frequent needs for quantitative reasoning skills in their jobs (Dumford & Rocconi, 2015). However, only about one in eight adults (13%) prove proficient in quantitative literacy (Elrod, 2014, para. 12), and even among college graduates, only one third demonstrated proficiency (Dumford & Rocconi, 2015, p. 1). Unfortunately, as even more recent data reveals, "Fewer than 10 percent of American students exhibit strong (level 5 or 6) QL skill" (Grawe, 2014, para. 4). Steen (2001) warned, "Despite years of study and life experience in an environment immersed in data, many educated adults remain functionally innumerate" (para. 4), and the current situation appears no better. Educators can no longer limit their instruction to isolated disciplines; the realities of the current workplace and global climate necessitate a broader perspective.

Defining Quantitative Reasoning

In the late 1980s, Cremin argued, “Americans were a more literate population at the end of the twentieth century than at its beginning” (Orrill, 2001, p. xiii). Literacy, Cremin (1988) concluded, cannot be defined only as technical skills in reading, writing, and computing; rather, “its meaning also depended on what an individual did with that technical skill, on how it was used . . . and to what ends” (p. 657). This emphasis on higher order application of more elementary techniques still frames the discussion of quantitative reasoning. The conception of a quantitatively literate individual spread throughout the late 1980s and significantly in the 1990s (Jordan & Haines, 2003). While the terminology is diverse, several common threads emerge to indicate the benefits of QR to the society and the individual outside of the traditional mathematics classroom.

Not just mathematics. Just as verbal literacy typically suggests more than the basic ability to recognize letter shapes and pronounce them in a linguistically appropriate manner, quantitative literacy entails more than numbers, symbols, and their manipulation. Clearly, quantitative reasoning must involve mathematical concepts. However, QR is not “just mathematics” (Elrod, 2014, para. 7). It involves the application of mathematics to a broader context (Dumford & Rocconi, 2015), the practical skills that accompany the academic discipline (Wismath & Worrall, 2015). Orrill (2001) described the contrast between the professional mathematician, who often sequesters himself or herself into the land of the abstract, and the numerate individual, who uses abstract mathematical ideas in various concrete settings. In particular, Orrill writes,

> This is not to say, of course, that mathematics and numeracy have little to do with one another . . . . [In a sense,] numeracy should be thought of as the extension of

> mathematics into other subjects in which, too often, the quantitative aspects of life are ignored altogether. (p. xviii)

Although the obvious connection with quantitative processes means much of QR education occurs within mathematics classrooms, the purpose of QR extends into all other areas, just as reading and writing prove integral in every discipline.

Quantitative reasoning and citizenship. Numeracy is not only an academic pursuit, as its value lies in its importance to and application within the broader society. In the late nineteenth century, ten collegiate educators worked to establish a precedent for curriculum that would provide the "fullest equipment for citizenship" (Mackenzie, 1894, p. 149; Madison, 2015, p. 11). Known thereafter as the Committee of Ten, they set some of the first widely accepted expectations for mathematics in higher education and defined the importance of applying mathematical skills to real-world experiences.

In the early 1980s, Cockcroft first defined numeracy as "the ability to cope confidently with the mathematical demands of life" (Wismath & Worrall, 2015, p. 1), demands that all students face even before they enter a career. Jordan and Haines (2003) emphasized the fast-changing needs of "an increasingly technological and quantitative world" (p. 16) that characterize modern employment and daily life, and the age of computers brings accelerated—often "bewildering"—change as numbers and data flood everyday experience (Orrill, 2001, p. xv). News and media, politics, economy and health issues, and the globally accessible marketplace require an ability to decipher and make decisions based on numerical information retrievable within fractions of a second. The quantitatively literate individual uses mathematical tools to understand, engage, and strengthen real-world situations.

Although certainly not the first of its kind in history, a movement toward extension beyond mathematical processes in the classroom—termed the "integrative phase" (Madison, 2015, p. 3)—began around 2000, after which grade schools and undergraduate institutions began to address more seriously the level of preparation they provided their graduates. Partially born from an increased awareness of and emphasis on practical workforce preparation, schools began to underscore not only the importance of "understanding of quantitative information" but also "the ability to use numerical, statistical, and graphical information in everyday life, as well as in the workplace" (Dumford & Rocconi, 2015, p. 1). The ancient academies instructed in order to equip citizens better for involvement in their societies. The trend over the last two decades has been for modern institutions to begin to provide better integration between what is typically isolated subject matter and real-world applications.

Quantitative reasoning and communication. Quantitative reasoning also provides facility in the language of mathematics, a language all people use but often without accuracy or precision. In 1997, Steen warned, "An innumerate citizen today is as vulnerable as the illiterate peasant of Gutenberg's time" (p. xxvii; Orrill, 2001). The ability to communicate—both to take in information and to reveal it to others, whether in the form of numbers, charts, graphs, or comparisons—proves essential to a growing global and connected community. Many scholars articulated the importance of QR as "a cultural field where language and quantitative constructs merge and are no longer one or the other, reflecting the continued suffusion of arithmetic with meanings" (Madison, 2004, p. 9). Numbers carry little value unless effectively communicated.

Quantitative reasoning and critical thinking. Not only does QR contribute to daily operation and communication, but skills in transferring mathematical ideas also allow individuals to interpret and analyze arguments and critically evaluate information. Jordan and Haines (2003) defined QR as "the ability to select, apply, and explain a variety of quantitative methods across different contexts" (p. 16-17), and Berg et al. (2014) included the "ability to apply quantitative skills to problem solving" (para. 2). Even elementary mathematics skills become powerful tools in thinking independently, evaluating and making informed decisions, and asking questions and confidently confronting experts (Elrod, 2014; Orrill, 2001).

Like most skills involving extended application of basic facts or techniques, QR requires practice. The ability to think using QR skills becomes a "habit of mind" (Madison & Deville, 2014, para. 9). Due in part to initiatives of Steen and Orrill since 2000, "QL is becoming accepted as an expected learning outcome of college" (Madison & Deville, 2014, para. 2). Increasing numbers of students now practice and apply quantitative skills as a required, supported piece of their formal education. Many higher education institutions continue to launch structured courses or programs designed to address the formation and practice of thinking using QR skills (Elrod, 2014; Madison & Deville, 2014). However, the success of such programs also lies with the preparation of incoming students in their primary and secondary school experiences.

Education Reform and Quantitative Reasoning

Smith and Thompson (2007) stated the obstacle teachers—especially mathematics teachers—encounter:

> For too many students and teachers, mathematics bears little useful relationship to their world. It is first a world of numbers and numerical procedures (arithmetic), and later a world of symbols and symbolic procedures (algebra). What is often missing is any linkage between numbers and symbols and the situations, problems, and ideas that they help us think about. (p. 3)

As more diverse groups of educators join mathematicians in placing emphasis on QR skills, new initiatives shape curriculum.

Primary and secondary standards. In recent years, K-12 education placed even greater attention on student numeracy. "The increasing need for QL, however, has outstripped those increases, at least cancelling the relative gain," noted Madison (2015, p. 1). In 2009, the Common Core State Standards for primary and secondary schools attempted to standardize expected outcomes for public education in English Language Arts and Mathematics (CCSSI, 2015a). The Common Core State Standards for Mathematics (CCSSM) aimed to provide students with "the knowledge and skills students need to be prepared for mathematics in college, career, and life" (CCSSI, 2015b, para. 1), a goal of prior standards, though more specifically stated and explicitly outlined than previous decades.

The CCSSM addressed the mathematical topics one would expect from mathematics curriculum (e.g., Counting and Cardinality, Measurement and Data, Expressions and Equations), but requirements encourage students to solve real-world problems (Madison, 2015), following in the direction of the QL movement. Because most mathematic skills needed for QR are at an elementary level, "many . . . conclude that QL is a K-12 issue rather than a collegiate issue" (Madison, 2004, p. 10). However, on account of the sophistication and the broad extensions required, many students do not become fully numerate before the end of their grade school years.

Pressure on higher education. On account of restricted time and resources, primary and secondary schools focus much of their attention on mathematical development toward college and career readiness. Even strides in pre-college standards, such as CCSSM, will take several years to evaluate and perfect, so institutions of higher education will likely need to wait for more formal measurements and improvements in QL (Madison, 2015). Therefore, the bulk of the accommodation for QR will, at least initially, come from colleges and universities recognizing the needs of the majority of their students in applying the mathematics skills beyond the mathematics departments.

Notwithstanding slow changes in lower levels of education, colleges and universities must step in for additional reasons. The AAC&U (2007) advocates for "progressively more challenging problems, projects, and standards of performance" (p. 3; Elrod, 2014). As Elrod (2014) pointed out, the integration and application skills required by QR fall at the top of Bloom's taxonomy of cognitive development. As a result, Wismath and Worrall (2015) noted, a distinction forms within post-secondary education between the academic discipline—traditional mathematics courses—and the application in real-world situations—quantitative reasoning. Earlier levels reflect the memorization and recall characteristic of primary and secondary mathematics, but colleges and universities that train students in evaluation and extension of information beyond basic levels must expect to do this equipping in quantitative areas, as well.

The two mathematics. While much of QR education will take place within institutions of higher education, at least initially, the place of QR within the curriculum remains debated. The conventional system assumes "QR is already taught in mathematics classes," but "experts argue that . . . most math courses don't teach QR skills" (Elrod, 2014, paras. 15-16). Research shows "taking one or more traditional math courses does not necessarily develop quantitative reasoning" (Agustin, Agustin, Brunkow, & Thomas, 2012, p. 312; Wismath & Worrall, 2015, p. 3). Madison (2004) discussed the dichotomy between the traditional mathematics course and QL mathematics: "With QL mathematics becoming both more demanding and more in demand. . . . the pressure on formal mathematics to respond with more effective QL education is increasing" (p. 10).

In 2007, the AAC&U included QL as an outcome on the Valid Assessment of Learning in Undergraduate Education (VALUE) rubric, guiding formation of QL education in many colleges and universities (Madison & Deville, 2014; Quantitative Literacy VALUE Rubric, n.d.). By promoting six criteria or areas of emphasis—interpretation, representation, calculation, analysis/synthesis, assumptions, and communication—the rubric provides helpful definitions in the development of QL courses (Berg et al., 2014).

While passing a formal mathematics course typically relies on displayed mastery of mathematical content or processes, strides in QR prove difficult to assess. As a result, existing mathematics courses must expand or additional courses must develop to incorporate QR into higher education curriculum. Nonetheless, most QL initiatives continue to occupy space almost exclusively within mathematical science departments or interdisciplinary learning centers, and "as of now, there are no established guidelines for

QL courses and no accepted, effective measures of long-term transfer" (Madison & Deville, 2014, para. 6).

Increasing Quantitative Reasoning in Undergraduates

Since the increased attention on QR in the 1990s, educators and organizations stressed the importance of integration with and transfer to other disciplines (Madison, 2015). Several universities piloted courses devoted to QR, while many others incorporated QR objectives and strategies into existing mathematics courses. As the successes and shortfalls of the English literacy movement reveal, though, limiting such a global outcome to a miniscule proportion of coursework will bring little success.

Across the curriculum. Questions remain concerning the place of QR in higher education in general and within particular mathematics and general education courses. As noted above, most experts view QR as separate from, though not unrelated to, traditional mathematics, therefore holding a different place in curriculum. "The very nature of QR is interdisciplinary because it involves contextual problem solving in real-world situations" (Elrod, 2014, para. 10; Hughes-Hallett, 2001). A similar concept in English literacy gave rise to "writing across the curriculum," promoting the instruction in and exercise of writing and reading skills in all core areas of education, including the less word-dense fields of science, mathematics, and the arts (Jordan & Haines, 2003).

Carefully constructed assignments in courses outside of the mathematical sciences promote QR while adding value to the outside courses (Berg et al., 2014; Orrill, 2001), just as many subject areas benefit from the clarity and creativity flowing from the inclusion of literacy skills education. Berg et al. (2014) called for the development of strategies allowing diverse departmental participation in teaching QR. Faculty outside of schools of mathematics will need both training and empowerment if the same "interdisciplinary communication" holds potential for QL: If mathematics faculty can learn and coach writing, literature faculty can do the same for QL (Madison, 2004, p. 11).

Transfer. Teaching QR within non-mathematical disciplines will also contribute to *transfer,* what Halpern and Hakel (2003) define as the purpose of formal education (Madison & Deville, 2014). Although content is meant to be useful to students, educators often only assume students will carry concepts and strategies to real-world situations. Educators use "repeated exposure to logically equivalent problems" in hopes that "children [will] distill the underlying reasoning schemes and develop meta-cognitive insights" that will transfer outside of school (Barnett & Ceci, 2002, p. 613).

However, especially within mathematics, the connection to real-world experience is difficult for students to make. In a study of the National Survey for Student Engagement (NSSE), Dumford and Rocconi (2015) reported, "Non-STEM majors tended to have more difficulty in formulating examples and [thinking] beyond mathematics courses" (p. 8). Studies concerning transfer resulting from formal education give little indication that much has occurred (Barnett & Ceci, 2002). Madison (2004) noted in particular that "learning mathematics for long-term transfer" is difficult (p. 10). Extending QR practice beyond the mathematics classrooms will increase the opportunity for formal higher education to transfer into authentic life experiences, especially because the limitations of mathematics courses cannot cater to all of the "unforeseen contexts required by QL" (Madison, 2004, p. 10).

Assessment. As institutions of higher education recognize the importance of QR within curriculum, they will continue to call for assessment tools and standards to improve instruction. In 2010, NSSE developed a set of experimental questions regarding student perceptions of their participation in activities known to researchers to connect with QR abilities. Surveys spanning two years included adapted and refined pilot questions, resulting in the three questions addressing QR that first appeared on the 2013 NSSE assessment (Dumford & Rocconi, 2015). Although the results do not measure actual QR abilities or levels of numeracy among students, the survey reveals “how often students report participating in QR-related activities” (p. 3).

Conclusion

Students transitioning into careers and lives post-college require not only mathematical skills but also the ability to apply them in the workplace, in media, and in interactions with others. The need for numeracy is comprehensive, and the onus of instruction falls largely on higher education. Although young when compared to the more traditional English literacy movement, the QR movement has made strides in definition and assessment and will continue to gain strength as educators from fields outside of mathematics contribute. By expanding the teaching of QR to span multiple curricular areas, colleges and universities provide students the opportunity to apply basic mathematical skills acquired in primary and secondary levels to their daily lives and careers.

Research presented in the following chapters contributes to the process of addressing the needs outlined by Dumford and Rocconi (2015). In particular, they called for further research to “fully explore the effectiveness of QR policies, courses, programs, and centers” in order to “investigate the link between involvement in QR activities with actual QR abilities” (p. 13). While data concerning engagement in QR activities has recently formally begun on a broad scale, longitudinal studies concerning the development and directions of QR education will allow for a more full exploration of whether new emphases in course curriculum have increased student engagement and accomplishment in QR abilities.

The following chapters outline the methodology and results from a qualitative study addressing trends in general education mathematics curriculum. Chapter 3 describes the tools and processes utilized. Chapters 4 and 5 present results and discussions of emerging themes and strength of trends.

Chapter 3

Methodology

This research examined the trends in higher education mathematics curriculum that foster increased attention to quantitative reasoning (QR). Employer and societal expectations often dictate crucial learning outcomes for college students, and higher education faculty adjust curriculum to address student needs. While state standards typically govern K-12 curricular structures, colleges and universities have more liberty to diversify instruction within the bounds of accreditation regulations. Therefore, colleges and universities vary extensively in their educational foci and approaches to teaching. As institutions recognize and respond to changes in expected outcomes of higher education in the realm of QR education, course subject matter and methods will reflect updated approaches.

Approach and Design

For the current study, the researcher employed a qualitative collective case study design to examine QR education in higher education mathematics curriculum development. According to Creswell (2012), case studies explore real-life, contemporary bounded systems. In this instance, a multisite study examined iterations of course catalog documents to explore changes or trends in general education mathematics course themes and descriptions.

A document analysis of published course or department catalog descriptions indicated the changes in curriculum for each institution. The study provided quantified descriptions of the presence of QR components for individual institutions, "followed by a thematic analysis across the cases" to assert overall themes (Creswell, 2012, p. 101). Analysis of trends in course development revealed whether colleges and universities have responded to various calls from their environment—future employers of graduates, researchers in the field of QR development, and the broader community—to update and expand curriculum to include engagement with quantitative information in real-world contexts.

Participants

The study examined a sample of the 50 top national liberal arts colleges, as defined by U.S. News & World Report (2015). Because they value student engagement with a broad curriculum, liberal arts institutions likely adopt new strategies and practices to promote stronger quantitative preparation for students as much or more often than other institutions. Eighteen institutions made course catalog documents for each year considered in the study available either online or through personal communication with registrar offices, which proved sufficient to reach data saturation[1]. All 18 institutions in the sample are categorized by the Carnegie Classification as more selective, four-year, baccalaureate colleges with an arts and sciences focus (Indiana University School of Education, 2015). Citations for consulted catalogs comprise Appendix A.

[1] Out of the nineteen institutions examined, one had no stated mathematical or quantitative requirement. As a result, it was not included in the statistical analysis, although it was still included in the remaining discussion.

Instruments

Individual faculty members often have the freedom to design syllabi to meet their departmentally or institutionally defined objectives. As a result, courses do not necessarily fit a particular design or mold. Assignments and assessments are often designed to fit the style and preferences of individual instructors and institutions. In 2013, the National Survey of Student Engagement (NSSE) incorporated a component to measure self-reported levels of student participation in QR activities in order to provide self-assessment data for individual colleges and universities and to facilitate comparison across multiple institutions (Dumford & Rocconi, 2015; National Survey of Student Engagement [NSSE], 2015). However, because the instrument is so recent, data do not yet determine whether institutions show consistent progress. In order to determine whether colleges and universities take seriously the call to increase QR skills in their graduates, educators should consider trends in course development.

The review of literature presented in Chapter 2 indicated several important characteristics of courses that promote QR development in students. This study examined academic course catalogs to determine whether institutions have implemented learning activities to address these characteristics. Accurate and effective course descriptions provide a summary of key ideas and aspects of a course. In order to determine whether significant instructional changes have occurred, the researcher for the current study examined academic catalogs from each institution for the 2002-03, 2003-04, and 2004-05 school years in addition to the more recent 2012-13, 2013-14, and 2014-15 catalogs. The span of a decade revealed existing trends, and the use of three consecutive catalogs ensured no data was missing (e.g., courses on a biannual rotation).

The researcher used the frequency (or lack) of QR related activities and themes in course descriptions to indicate the level of emphasis on QR within the course. Specifically, one or more of the following in a course description indicated the presence of QR outcomes:

1. Explicit mention of *quantitative reasoning*, *quantitative literacy*, or *numeracy* skills;
2. Emphasis on *analysis* or *synthesis* of quantitative information (Madison, 2014; Madison, Boersma, Diefenderfer, & Dingman, 2011; NSSE, 2015);
3. Examination of *real-world* (*real-life*) *problems* or *context* (Berg et al., 2014; Dumford & Rocconi, 2015; Madison, 2004, 2014; NSSE, 2015; Orrill, 2001); and
4. Presence of *making judgments* or *evaluating arguments* based on quantitative information (Elrod, 2014; Madison et al., 2011; NSSE, 2015).

Procedures

The present researched did not involve any identifiable risk with the acquisition or analysis of course catalog documents, thereby removing the need to contact institutional review boards of participant institutions. The researcher obtained institutional course catalogs through individual institution websites or institutional registrar offices. The researcher also identified courses satisfying general education requirements in mathematics. Next, the researcher examined course descriptions for the presence of QR themes, both explicit and implicit, as indicated above, and entered data into a Microsoft Excel spreadsheet to facilitate trend analysis.

Analysis

For each institution and catalog, the researcher recorded the description of courses required in mathematics—or comparable quantitative requirement—in addition to the number of courses that fulfill the requirement. The researcher also denoted the frequency of QR themes, based on the presence of the four items listed above, for each course. The researcher then determined average frequencies by dividing the total number of courses in which any of the four items appeared by the number of descriptions analyzed for each institution within three-year intervals (i.e., 2002-03/2003-04/2004-05 and 2012-13/2013-14/2014-15). The researcher employed a paired *t*-test to compare the frequency of QR themes in the three earlier catalogs to the latter catalogs to determine whether significant change has occurred. Chapter 5 states and further discusses trends in course descriptions as well as notable themes in general education requirements.

Benefits

The call for development of new curriculum to address the changing needs of graduates saturates higher education literature. This study determined whether recommendations by experts have made their way into actual practice in college and university curriculum. Institutions hoping to increase student engagement in QR activities benefit from description and analysis of a sample of highly effective institutions as examples to follow or compare.

Chapter 4

Results

The process of gathering, interpreting, and analyzing course descriptions from a wide variety of schools yielded a broad array of institutional adaptations and incorporations of new educational programs. Some institutions modified individual courses to educate better for numeracy. Because the process to reform the curriculum takes time, even small changes indicate significant effort and energy in reshaping a course or its content. Other institutions restructured requirements to narrow the focus toward specific QR skills. Data collected reveals both of these trends, as well as more general shifts in communication of the philosophy behind core requirements. Examination of college and university general education requirements as well as individual descriptions of courses which fulfill them reveal institutions continue to turn attention toward quantitative reasoning outcomes in their curriculum.

Course Description Analysis

Aside from personal testimonies and individual syllabi, published course descriptions prove the most succinct summary of the content and/or goals of a course. A review of more than 1,500 course descriptions from eighteen institutions offered a snapshot of the emphasized topics among participant institutions. Table 1 provides a summary of the collected data, the weighted means for QR themes present in required courses for each three-year span, and the difference in means over the course of the study for each institution. A negative difference indicates a decrease in the average presence of QR themes among courses. (For a full data presentation, including the number of courses for each academic catalog which qualify within the core requirement and the number of courses that contain one of the four QR themes given in Chapter 3, see Appendix B.)

Table 1

Proportion of Mathematics General Education Courses Containing QR Themes

Institution	Proportion of Courses Fulfilling Requirements in Which QR is Present		Difference in Means (μ_d)
	2002-2005 (μ_1)	2012-2015 (μ_2)	
Bard College	0.1111	0.1667	0.0556
Bowdoin College	0.2206	0.4154	0.1948
Carleton College	01918	0.7895	0.5977
Colgate University	0.1667	0.1685	0.0019
Davidson College	0.0000	0.2895	0.2895
Hamilton College	0.1667	0.4167	0.2500
Kenyon College	0.1111	0.1875	0.0764
Macalester College	0.2667	0.3108	0.0441
Occidental College	0.3778	0.2593	**-0.1185**
Skidmore College	0.3846	0.2692	**-0.1154**
Soka University of America	0.4000	0.4286	0.0286
University of Richmond	0.1667	0.5000	0.3333
Vassar College	0.2000	0.2500	0.0500
Washington and Lee University	0.0000	0.1667	01667
Wellesley College	0.3750	1.0000	0.6250
Wesleyan University	0.0508	0.0167	**-0.0342**
Whitman College	0.2545	0.2778	0.0232
Williams College	0.3924	0.2471	**-0.1453**

Note. Negative differences in means, representing a decrease in permeation of QR themes, are in boldface.

While a table of means may start to reveal tendencies, a statistical significance test translates these proportions into meaningful information. Because the sample is not highly skewed, a paired t-test can provide a strong indicator for significance even utilizing a smaller sample size (n=18). The paired t-test is a regular t-test performed on the difference of two correlated means, in this case, the mean from a decade ago and the more recent mean. Significance testing determines "whether a sample statistic [e.g., mean] is 'significantly' far from what would be expected if the null hypothesis were true in the population" (Utts & Heckard, 2007, p. 522). The null and alternative hypothesis for the test are as follows

H_0: $\mu_d = 0$ (Or, there is no change in average presence of QR themes from the first set [2002-2005] to the second set [2012-2015] of catalogs.)

H_a: $\mu_d > 0$ (Or, there is a positive change in average presence of QR themes form the first set [2002-2005] to the second set [2012-2015] of catalogs.)

Because the research only addressed an increase in the amount of QR appearing in courses (as opposed to QR becoming less common), the alternative hypothesis remains one-sided. Table 2 summarizes the relevant values for conducting a t-test.

Table 2

Summary of Descriptive Statistics for Institutions Sample, n = 18

Sample Size, n	Degrees of Freedom, df	Sample Mean	Standard Deviation	Standard Error	T-statistic	P-value
18	17	0.1291	0.2217	0.0538	2.4005	0.0144

Based on the test statistics generated using Microsoft Excel, the resulting p-value for the data set is $p = .0144$. Therefore, the null hypothesis can be rejected. Significant evidence for a positive change in QR presence in mathematics general education courses among the population.

Types of change. As shown, highly rated liberal arts colleges and universities have generally accomplished gains in QR prevalence in their courses. Institutions made changes in a variety of ways, however. Updating individual courses, adding new ones, or restricting the qualifying courses to a subgroup each produce similar results in educating students with a higher QR focus.

Limited selection. For some institutions, the increase came from more specific requirements, limiting the number of options students have to a smaller set of courses more likely QR-focused. Often, this change resulted from the adoption of a more specific quantitative requirement. In the interim between 2005 and 2012, Carleton College refined its expectations from a general Mathematics and Natural Sciences requirement with more than 20 sufficient courses to a more narrowly-defined Quantitative Reasoning Encounter, fulfilled by only six or seven courses offered in a given year. Davidson College (2012a) also reduced the qualifying courses from approximately 20 in the earlier years to only 13. Similar to Carleton, the earlier requirement of any course in mathematics was refined to one of selected courses oriented toward Mathematical or Quantitative Thought.

New… Some institutions also added new courses. Hamilton University updated their quantitative requirements sometime after 2005 to include several more classes, most of which include some element of QR. Compared to the earlier course descriptions, in which only the introductory level statistics course includes application and analysis of quantitative information, later catalogs offered multiple courses including QR. Courses in Vector Calculus and an Introduction to Optimization, both featuring applications to science, engineering, economics, and other areas, recently came to fulfill Hamilton's

Quantitative and Symbolic Reasoning requirements, and an additional Statistical Analysis of Data course incorporates additional aspects of QR (Hamilton College, 2012b).

Bowdoin College, already with above average envelopment of quantitative literacy outcomes in older catalogs, made impressive gains over the decade of the study. The number of courses remained almost the same, but new courses replaced existing ones to emphasize new topics. For example, Bowdoin initiated a course entitled Quantitative Reasoning to address many of the themes discussed in this study. It also introduced a Biomathematics course to tie mathematics more seamlessly to its applications in the biological sciences.

....And improved. Still other schools, like Wellesley College and University of Richmond, kept similar requirements but updated the descriptions (and, presumably, course content) to include more specific attention on QR. For example, each of the three more recent Wellesley College catalogs includes application to real-world situations in the description of a Probability and Elementary Statistics course. The difference is clearly demonstrated here:

> Topics selected from the theory of sets, discrete probability for both single and multivariate random variables, probability density for a single continuous random variable, expectations, mean, standard deviation, and sampling from a normal population. (Wellesley College, 2004, p. 117)
>
> This course is about the mathematics of uncertainty, where we use the ideas of probability to describe patterns in chance phenomena. . . . Probability is the basis of statistics and game theory, and is immensely useful in many fields including business, social and physical sciences, and medicine. The first part of the course

> focuses on probability theory (random variables, conditional probability, probability distributions), using integration and infinite series. The second part discusses topics from statistics (sampling, estimation, confidence interval, hypothesis testing). Applications are taken from areas such as medical diagnosis, quality control, gambling, political polls, and others. (Wellesley College, 2012a, “MATH 220”)

The updated course description provides more description not only of subjects in the course but also some context for situations in and out of mathematics where the topics are relevant.

Similarly, the University of Richmond more recently included applications to other fields in its course descriptions for Calculus II and Scientific Calculus II. The following comparison comes from two descriptions of Calculus II:

> Techniques of integration; applications of integration; improper integrals; l’Hospital’s Rule; Taylor’s Theorem and applications, infinite series, differential equations. (University of Richmond, 2004, p. 125)
>
> Techniques of integration; applications of integration; improper integrals; Taylor's Theorem and applications; infinite series; differential equations; applications to the sciences, social sciences, and economics. (University of Richmond, 2012b, “MATH 212”)

While neither cases guarantees a greater grasp of QR by students, the shift in language certainly indicates efforts by faculty members and their respective universities to incorporate new updated content.

Not always positive. Despite the overall trend in increased QR inclusion, not every institution made positive changes. Four analyzed institutions—namely Wesleyan University, Skidmore College, Williams College, and Occidental College—seem to have decreased in numeracy emphasis. A variety of factors could explain the negative trend (e.g., in Skidmore's case, a decrease in courses fulfilling the foundational requirement eliminated some courses that contained QR themes). Notably, though, of the four institutions, none have made significant changes to the requirements or description of their general education program in the duration of the study.

Centre College, unlike the other institutions in the sample, has no definitive quantitative requirement in its core program. Since the 2002-2003 academic year, students had only to gain "further fluency" in either mathematics, computer science, or a foreign language. Consequently, the researcher did not include Centre in the statistical analysis detailed above but in the presentation of results for the sake of due diligence.

Themes present. Despite the variety in forms of change, the actual content of the change appears much less diverse. Not surprisingly, due to the influence of the "usefulness" strand in mathematics education, much of the new and maintained focus related to numeracy tends toward utilizing quantitative procedures and information in real-world contexts. Applications in the physical and biological sciences, economics, population dynamics and political realms, and medicine permeate course descriptions. In fact, out of 395 courses containing at least one of the four QR elements in this study, 268 are marked only for *examination of real-world (real-life) problems or context*. The second most common theme, related to recent priority on critical thinking in recent years, comes in the *emphasis on analysis and synthesis of quantitative information*. Far less common are *explicit mentions of quantitative reasoning, quantitative literacy, or numeracy* and *making judgments or evaluating conclusions based on quantitative information*. Only 25 courses included either of these outcomes at all.

Additional observations. Aside from courses incorporating or emphasizing typical QR elements, "pure mathematics" remains the norm. Pure mathematics studies abstract concepts of the field. The calculus sequence, higher order algebra and geometry, analysis (which has a specific meaning in mathematical context), among other topics constitute most of the courses offered, echoing the GATC tracks in secondary education.

However, several institutions incorporate courses into their curriculum not fitting within common pure mathematics or even more progressive quantitative literacy categories. Often, these courses contain historical or cultural perspectives on mathematics, examining the development of mathematic ideas through different cultures and individuals. At Kenyon College, a History of Mathematics in the Islamic World course, first observed in the 2012-2013 catalog, "examines an important and interesting part of the history of mathematics, and, more generally, the intellectual history of human kind: the history of mathematics in the Islamic world," emphasizing history, social sciences, and religion as well as mathematics ("MATH 128"). Another present—though certainly not widespread—and intriguing subject in and rationale for mathematics core curriculum is beauty. Neither fits nicely into the quantitative reasoning category as defined, but they do not stay within the confines of pure mathematics courses, either.

General Education Descriptions

Course descriptions reveal increases in QR outcomes, but the philosophy behind the requirement in the first place exposes even more. Indeed, on account of limited catalog space, course descriptions cannot completely express everything a course entails. In addition to changes and expansion in QR themes in courses satisfying general education requirements in the mathematical sciences, several institutions had existing descriptions incorporating QR or made observable changes in wording of their mathematical or quantitative requirements.

Responding and revising. Updated catalogs indicate dynamicity among higher education institutions. In order to ensure students graduate well prepared by their liberal education, high-achieving liberal arts institutions have begun to make changes in alignment with current research and trends. Eight institutions made significant changes to their catalog descriptions of their mathematical or quantitative requirement over the twelve-year course of this study.

Three institutions (Vassar College, Washington and Lee University, and Whitman College) changed only the description of the quantitative requirement. Vassar College, for example, through 2004-2005, had a Quantitative Course requirement. “Numeracy, like literacy, is important in a liberal education,” the catalog plainly states (p. 43). By 2012-2013, the description expanded:

> Facility in quantitative reasoning is an important component of liberal education. Quantitative reasoning includes the ability to understand and evaluate arguments framed in quantitative or numerical terms; to analyze subject matter using quantitative techniques to construct and evaluate quantitative arguments of one’s own; and to make reasoned judgements about the kinds of questions that can be effectively addressed through quantitative methods. (p. 25)

Although Vassar's qualifying courses and descriptions stayed mostly the same (a few extra courses satisfy the requirements in the later years), the institution's increased attention to QR proves evident.

Whitman College also made significant changes to its rationale for distribution requirements in quantitative areas. Students entering in or after the fall of 2002 took a course in Quantitative Analysis, for which any mathematics course sufficed (p. 37). (Prior to 2002, students only took six credits in either the physical sciences or mathematics.) Catalogs in and after the 2011-2012 academic year still require one quantitative course, for which all mathematics courses apply. However, a description is included to reveal the rationale for such a course:

> Courses with a significant quantitative focus help us to develop the skills to critically analyze numerical or graphical data, to develop abstract quantitative frameworks, and to develop facility and acumen with quantitative reasoning techniques and their applicability to disciplines across the liberal arts. (Whitman College, 2012, p. 44)

Several QR themes now appear, suggesting a more targeted approach to educating for numeracy.

Three colleges—Bowdoin College, Carleton College, and Davidson College—moved from more general distribution requirements in the mathematical or natural sciences to more specific mathematical reasoning or thought requirements. Bowdoin's Natural Science and Mathematics distribution requirement (Bowdoin College, 2004, p. 25) was replaced by a Mathematical, Computational, or Statistical Reasoning distribution requirement before the 2012-2013 catalog. Where further description was lacking a decade earlier, students are now "enabled to use mathematics and quantitative models and techniques to understand the world around them either by learning the general tools of mathematics and statistics or by applying them in a subject area" (p. 17). Finally, Macalester College, keeping its initial distribution requirement in Natural Sciences and Mathematics, added a Quantitative Reasoning requirement in the interim of this study.[2]

Status quo or maintaining focus? Several institutions made no major changes to their general education descriptions in mathematics or quantitative subjects, but their initial requirements already reflected a focus on QR. Williams College (2012), for instance, had a Quantitative/Formal Reasoning requirement since the Class of 2006 entered the college. For at least 12 years, the requirement "intended to help students become adept at reasoning mathematically and abstractly" and to develop "the ability to apply a formal method to reach conclusions, to use numbers comfortably, and to employ the research tools necessary to analyze data" in order to prepare students better for future professional roles (p. 10). Kenyon College also maintained a Quantitative Reasoning expectation since 2002-2003, requiring courses that "may focus on the organization, analysis, and implementation of numerical or graphical data; or they may involve learning mathematical ideas, understanding their application to the world, and employing them to solve problems" (n.p.). Further explanation includes all three additional aspects of QR looked for in course descriptions—clearly a thoughtful response to QR.

Similar to the institutions above that have not increased focus toward QR outcomes in course descriptions, four institutions offer general education requirements

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[2] Unfortunately, a listing of courses that fulfill the QR requirement is available only for the current semester on the registrar's website. As a result the Natural Sciences and Mathematics distribution requirement was used for both iterations of catalog analysis in this research.

not closely related to QR themes—and that did not change over the 12-year period of the study. Two (Occidental College and Wesleyan University) experienced a decrease in QR, as discussed above. The others, Colgate University and Soka University of America, saw two of the smallest positive gains in the sample. Further examination of these institutions and their broader curriculum could reveal increased attention to QR through other means, but any advancement did not prove visible in the scope of this study.

Published course catalogs, as described, do offer a limited perspective on institutional and departmental values and initiatives regarding QR education. This study showed some progress in adapting course descriptions and content to align more closely with recent research and students' and employers' needs. Overall trends indeed pointed to increased QR activities and outcomes in general education at high-achieving liberal arts institutions. Colleges and universities hoping to prepare numerate graduates better, though, still have opportunities for improvement. Chapter 5 further discusses implications of the findings on practice across institutional communities. It also presents suggestions for future research in the field of mathematics general education curriculum.

Chapter 5

Discussion

A survey of highly successful liberal arts institutions revealed subtle to substantial shifts in curriculum relating to quantitative reasoning. Even small changes, however, may reflect significant developments in values and practices impacting student outcomes. In some areas, modern mathematics returns to its historical roots, offering a full, freeing applicability to the natural world. In others, it remains bound to patterns and procedures relegating the subject only to what proves useful or pure. Like the ancient philosophers, mathematicians today call for connections across curriculum subject areas and for the use of mathematics to communicate meaningful conclusions. The separation of the field into discrete topics continues to receive resistance as institutions address students' real or perceived inability to utilize quantitative data in real-world contexts.

Reflecting the 1980s' desire to use mathematics to "cope confidently with the mathematical demands of life" (Wismath & Worrall, 2015, p. 1), much of the variation found in this study resulted from emphasis on application to real-world contexts and problems. Institutions' relationships with employers through internships and graduate placements no doubt drive this trend even as a larger understanding of mathematics' place in civic life integrates economics and social dynamics into more abstract dimensions of the field. Still almost completely absent, however, is attention to making or evaluating judgments based on quantitative information, which reflects poor extension of mathematics into the realm of effective communication. Innumeracy remains a threat; students practice mathematical syntax in specific situations but often lack the expectation and ability to extend the tools to narratives and arguments on a larger scale.

Changes in language and content of academic catalogs, though, show the potential for development in the area of QR education. For example, consider the prevalence of language now used to describe the purpose of a quantitative requirement compared to even ten years prior. From the institutional to the course level, academic catalogs reveal reforming and reshaping. Still, educators should not assume edits in formal language directly translate into expanded student outcomes. This and other limitations impact the implications of the study but also grant opportunities for future research.

Limitations of the Study

In order to achieve breadth across so many highly ranked institutions, the researcher drew descriptions and rationales for general education mathematics from official course catalogs. Institutions design these documents to give students accurate pictures of the purpose and content of particular courses. However, course catalogs do not—and cannot—contain all a college or university values, teaches, or offers. Furthermore, in order to compensate for diversity in organization and presentation, the researcher selected the particular methodological approach in order to look for changes in particular forms (e.g., specific terms or language) rather than to examine themes arising from a collection of data. Familiarity with mathematics education theory and practice suggests more QR is present and taught in courses than the catalog descriptions explicitly state. Given the nature of this study, however, the researcher could only assess what was written, leaving evidence somewhat restricted.

The scope of the data, although representative of high-ranked liberal arts institutions, also suffers from a lack of diversity across institutional type. Liberal arts institutions hold similar foundational values—the core liberal arts experience. Other institutions—especially STEM institutions—emphasize and operationalize mathematics requirements differently, so generalizability proved limited.

Finally, meaningful comparisons are lessened when requirements and sufficient courses fall into overly diverse categories. While most colleges and universities allow students to count any of many mathematics courses for their quantitative requirement, realistically the majority of students only take base-level courses. The few students who take upper-level courses fulfilling requirements have already completed prerequisites accomplishing the same purpose. Therefore, upper-level course options for general education, often containing fewer QR themes, prove relatively superfluous. Research could not take into account the actual courses the majority of students used to fulfill requirements, but weighing conclusions toward courses more commonly taken would provide a fuller picture of the general population's experience.

Implications for Practice

Exploration of the phenomena, however, still provides educators with suggestions for action. In order to provide for the needs of students and the desires of employers and communities, institutions need not only pay attention but also respond. Notably, the only institutions that did not increase in QR presence over the ten-year period also made no changes to requirement descriptions. Change without purpose may not lead to better outcomes, but neither will outcomes change without intentionality. Even institutions maintaining an emphasis on pure mathematics must acknowledge the transition students now make from common QR standards in elementary and secondary education. Mathematics faculty, as well as academic services, should examine best practices and prepare to care for a new generation of students' academic needs. To begin, they must recognize that students entering pure math courses may struggle to embrace the less-emphasized theoretical aspects of the material. In addition, they must take advantage of and build on students' increased knowledge in QR areas, further honing skills in analyzing, applying, and synthesizing quantitative information.

Furthermore, institutions hoping to incorporate more QR into their courses should consider tools like the AAC&U's Quantitative Literacy VALUE Rubric. The VALUE rubric offers summaries of student learning from basic to "capstone" in areas of interpretation, representation, calculation, application, making assumptions, and communication (Quantitative Literacy VALUE Rubric, n.d.). Especially for institutions heavy in certain aspects of QR (e.g., examination of real-world problems or contexts), the benchmarks can offer direction for shaping activities in a variety of performance areas.

Most mathematics courses naturally incorporate elements of interpreting, manipulating, and disseminating data. However, a purposeful communication of the value and extension of these skills will contribute even more to QR objectives. Even pure mathematics courses—retaining their emphasis in the abstract—offer opportunities to communicate theories clearly, analyze proof assumptions and conclusions, and translate numerical information from one form to another—all skills that reinforce QR.

At the same time, departments outside of mathematics must also bear some responsibility for increasing QR engagement. Educators should creatively consider which subject areas have information to interpret, conclusions to draw, assumptions to evaluate, and therefore the potential for numerate engagement. Although not the subject of this study, courses outside of mathematics often fulfill a quantitative requirement at many colleges and universities—an achievement to become recognized and broadened in many cases. In return, though, mathematics departments should also enhance courses to comprise other subject areas and even institutional learning objectives (e.g., literacy outcomes, civic engagement, or ethical development). Transmitting mathematics into other departments does not offer the only way to display the applicability of mathematics to real-world contexts and connectedness of subjects in a liberal arts curriculum. Including other subjects in mathematics curriculum accomplishes the same goals.

Implications for Future Research

As institutions refine their practice in regard to QR education, further studies could reveal the extent of responsiveness to new curricular demands as well as whether enhancements contribute to significant change in student learning in the intended areas. Methodologically similar studies at non-liberal arts institutions could expose similar or distinctive trends in curriculum development. Considering diverse approaches to QR education at research institutions, community colleges, STEM-focused institutions, and others may even provide best practices applicable across the field of higher education.

Interviewing mathematics faculty members and department leaders could also offer an expanded perspective on the scope of and rationale behind changes or the lack thereof. At institutions where significant shifts occurred, what inspired the change? Perhaps regular curriculum reviews motivated more modern language, or an intentional focus on QR drove innovation. If few developments took place, why? Formal academic documents may not reflect the actual values or status of an institution, or QR could be a minimal focus despite recent trends. Although veteran educators may offer greater context for these questions, students also serve as stakeholders in the decisions and could offer valuable insights concerning their needs.

Additional research could consider whether an explicit (or implicit) emphasis on QR activities translates into actual increase in QR abilities. Academic catalogs, institutional leaders, and even instruments like NSSE offer insight into the types and levels of engagement in activities thought to increase QR ability. Research could explore the results of engagement, pairing prevalence of QR activities with students' abilities to grasp and extend the skills. The Quantitative Literacy and Reasoning Assessment (QLRA), developed by Eric Gaze of Bowdoin College and a team of QR researchers and educators, seeks to provide the latter. Intended to provide a national database for institutional comparisons, the instrument measures students' ability to apply quantitative skills to solve problems (Gaze et al., 2015).

Finally, as the purpose of mathematics becomes once again debated and delineated, the place of topics falling outside of traditional pure and more QR-focused mathematics should receive further exploration. Historical and non-Western cultural approaches to mathematics offer broader views of the development and value of the subject, including diverse ways to approach and solve familiar problems. Putting mathematics in context, students could learn deeper, transfer more effectively, and better retain quantitative learning.

Beauty in mathematics also warrants investigation and discussion. Indeed, course descriptions at several institutions studied (e.g., Bard College and Wesleyan University) already mention the beauty of the field. Student interest and motivation to pursue mathematics from these alternate perspectives might encourage further (or for many, initial) investment in what often proves one of the least popular areas. More importantly, though, mathematics often simply becomes accepted as necessary because of its utility in working successfully and thinking critically: What if it rather became embraced as valuable for appreciation, enjoyment, and flourishing?

Looking Forward

From its beginnings as an integrated language to describe the world, to its prominence in a technology-filled society, to its development as a culturally useful and required skill—mathematics remains a central tenet to educational systems. A sample of highly ranked liberal arts institutions confirms attention toward the subject still today. Significant changes in mathematics general education curriculum since the turn of the century not only indicate responsiveness to increasing needs and demands but also reveal areas in need of further development.

As education and society continue to pressure mathematics to serve new functions, both limited and unlimited, collegiate mathematics educators have the potential to equip or to hinder. They may support students entering with various levels of preparation and challenge them to learn deeper, or they may persist in traditional means of educating, for better or worse. They may build appreciation for the subject, or they may perpetuate a distaste and accepted ignorance. Regardless, the need for mathematics—for understanding physical properties, for communicating meaningful changes, for approaching daily tasks, even for purely appreciating diverse fields of knowledge—remains.

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Appendix B

Supplemental Data

Table 3

Courses Fulfilling General Education Quantitative Requirements

Institution	Number of Qualifying Courses (Courses with QR Themes)					
	2002/03	2003/04	2004/05	2012/13	2013/14	2014/15
Bard College	16 (2)	15 (2)	14 (1)	20 (4)	19 (4)	21 (2)
Bowdoin College	22 (5)	23 (4)	23 (6)	21 (9)	25 (9)	19 (9)
Carleton College	27 (5)	21 (5)	25 (4)	6 (4)	7 (6)	6 (5)
Colgate University	24 (4)	24 (4)	24 (4)	29 (5)	30 (5)	30 (5)
Davidson College	19 (0)	21 (0)	20 (0)	12 (3)	13 (4)	13 (4)
Hamilton College	4 (1)	5 (1)	3 (0)	9 (4)	9 (4)	6 (2)
Kenyon College	20 (4)	22 (1)	21 (2)	32 (6)	32 (6)	32 (6)
Macalester College	21 (5)	18 (7)	21 (4)	28 (9)	23 (6)	23 (8)
Occidental College	19 (7)	13 (5)	13 (5)	27 (7)	27 (7)	27 (7)
Skidmore College	13 (5)	13 (5)	13 (5)	8 (2)	8 (2)	10 (3)
Soka University of America	5 (2)	5 (2)	5 (2)	7 (3)	7 (3)	7 (3)
University of Richmond	6 (1)	6 (1)	6 (1)	6 (3)	6 (3)	6 (3)
Vassar College	5 (1)	5 (1)	5 (1)	7 (2)	7 (2)	6 (1)
Washington and Lee University	6 (0)	6 (0)	6 (0)	7 (1)	5 (1)	6 (1)
Wellesley College	2 (1)	3 (1)	3 (1)	3 (3)	3 (3)	3 (3)
Wesleyan University	17 (0)	20 (1)	22 (2)	19 (0)	20 (1)	21 (0)
Whitman College	19 (3)	17 (6)	19 (5)	16 (5)	18 (3)	20 (7)
Williams College	26 (9)	27 (11)	26 (11)	29 (8)	26 (5)	30 (8)
Total	271 (55)	264 (57)	269 (54)	286 (78)	285 (74)	286 (77)